UNIT 2: TWO VARIABLE DATA

WHAT IS OUR GOAL FOR UNIT 2?

- **Representing** Relationships Between Bivariate Categorical Data
- **Representing** Relationships Between Bivariate Quantitative Data

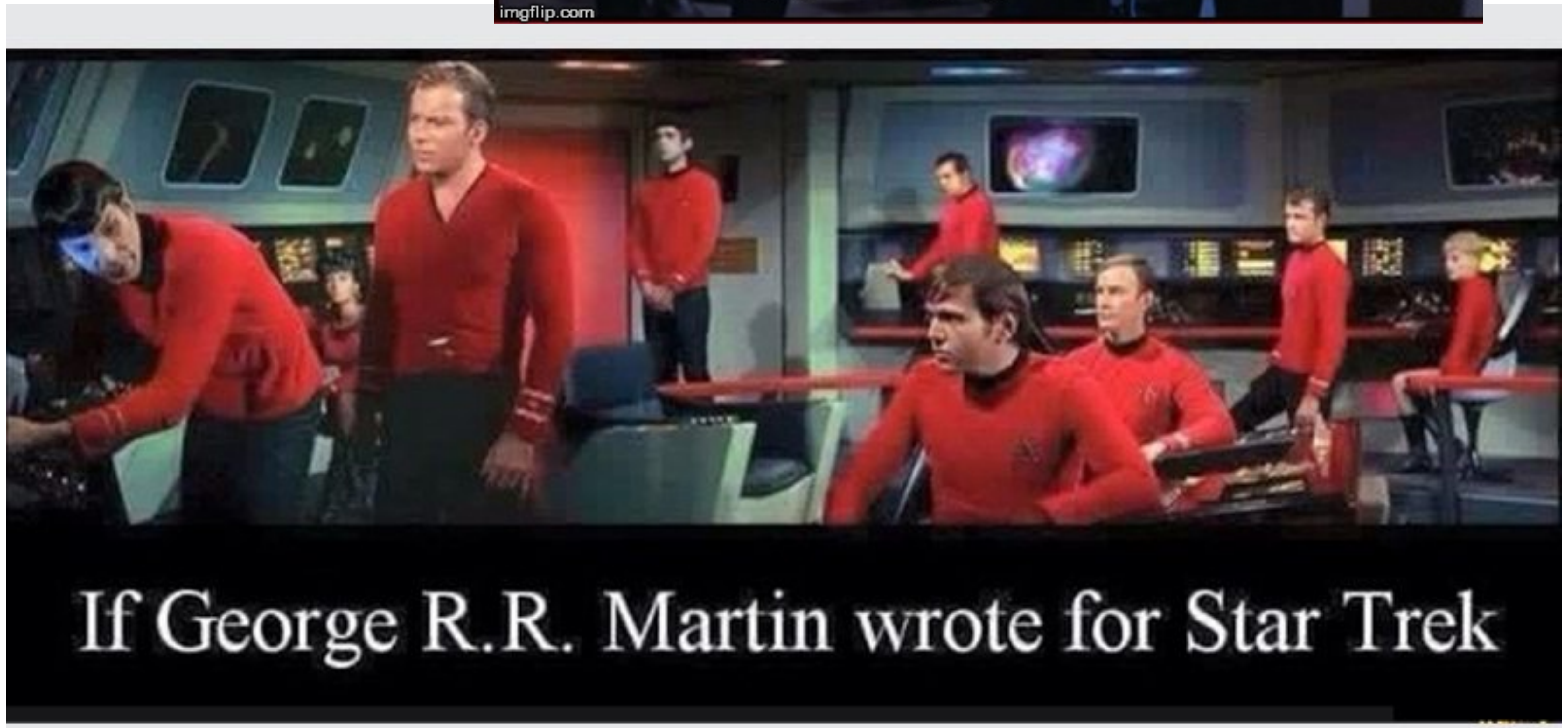
BIVARIATE CATEGORICAL DATA

- Is there a relationship between two Categorical Variables?
- We will represent relationships using **tables** (same as treat example before), **Graphs**, and **Statistics** (numbers)

BIVARIATE CATEGORICAL DATA

- **Our Example:** X: Shirt Colour 🔴 🟡 🔵 , Y: Status ☠️ 😃

Matthew Barsalou published an article in *Significance* that studies this from a statistical perspective



BIVARIATE CATEGORICAL DATA

- **Our Example:** *X*: Shirt Colour 🔴🟡🔵 ,   *Y*: Status ☠️ 😃

Crew Member	Area	Shirt Color	Status
Brendan	Operations, Engineering and Security	Red 🔴	DEAD ☠️
Leif	Command And Helm	Gold 🟡	DEAD ☠️
Shailah	Science and Medical	Blue 🔵	Alive 😃

Dataset is composed of 430 cremates

Enterprise NCC 1701 casualties from episodes aired between September 8, 1966 and June 03, 1969 based on casualty figures from Memory Alpha.

BIVARIATE CATEGORICAL DATA

- First we tabulate data into a **contingency table** (also known as a two way table)

	😃	☠️	
🔵	129	7	136
🟡	46	9	55
🔴	215	24	239
	390	40	430

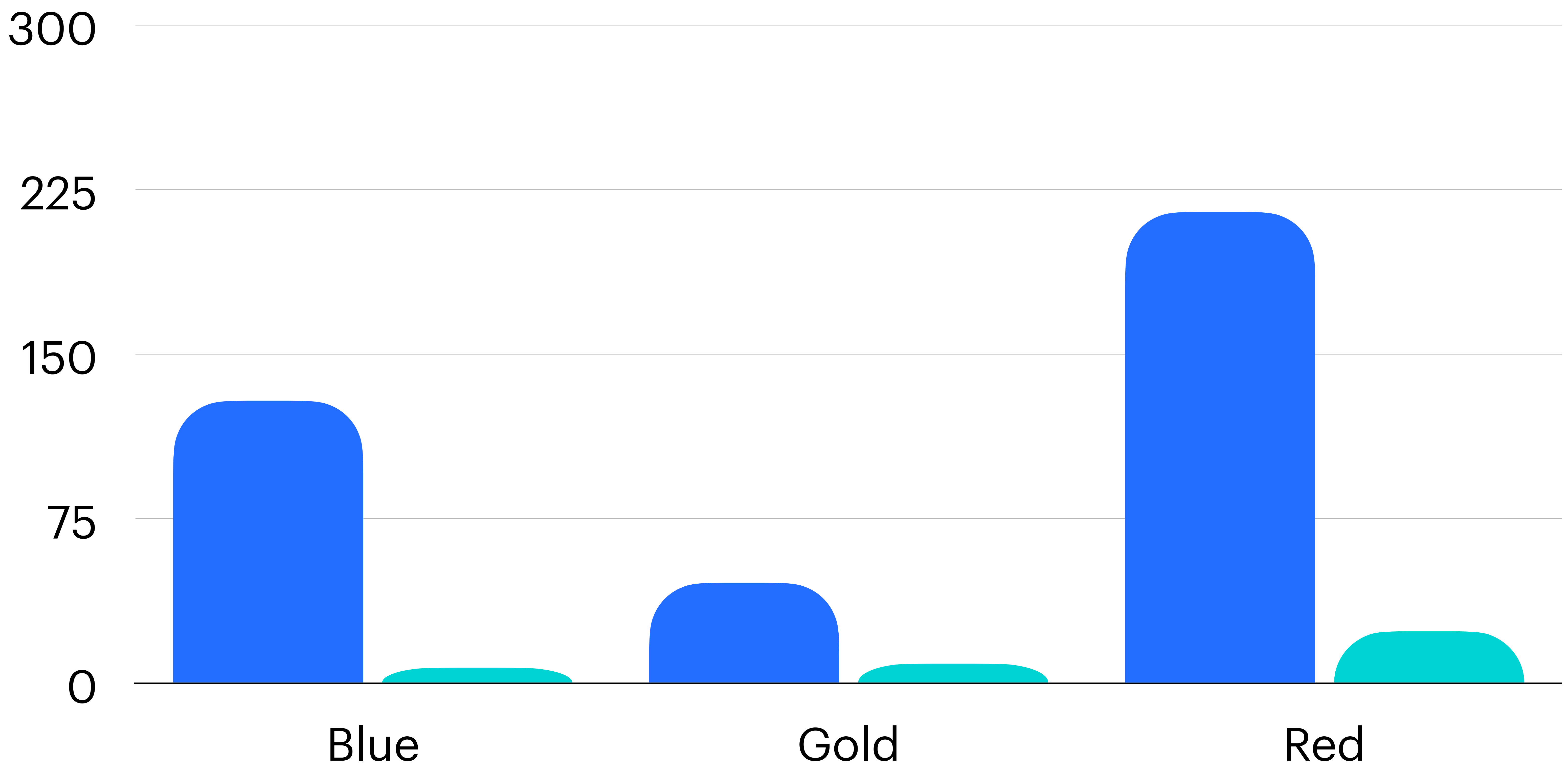
- **Marginal Distribution**
- **Joint Distribution**

BIVARIATE CATEGORICAL DATA

- First we tabulate data into a **contingency table** (also known as a two way table)

	😃	☠️	
🔵	129	7	136
🟡	46	9	55
🔴	215	24	239
	390	40	430

It’s hard to notice association when using frequencies



BIVARIATE CATEGORICAL DATA

Conditional Probability

	😃	☠️	
🔵	129	7	136
🟡	46	9	55
🔴	215	24	239
	390	40	430

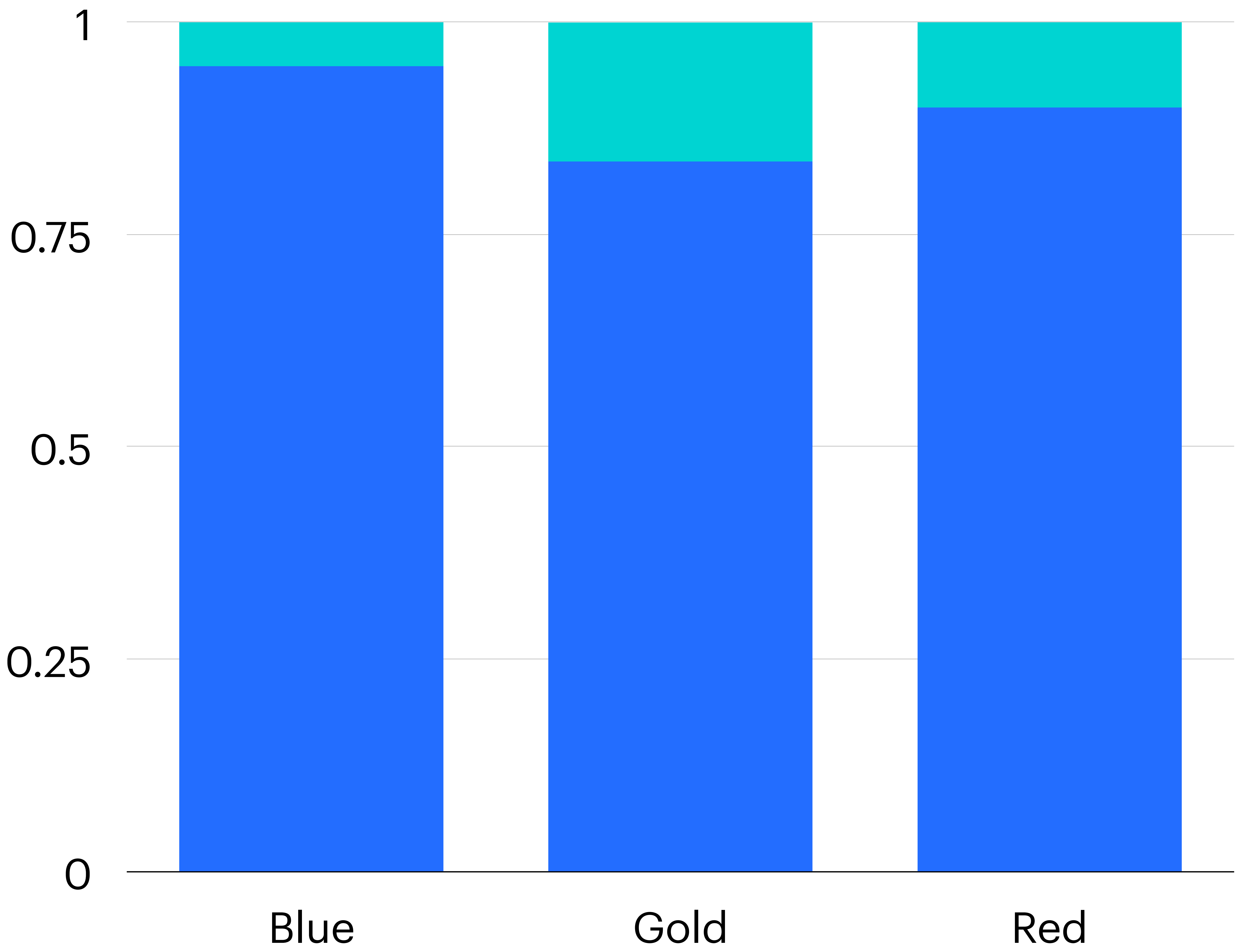
Questions

1. What is the probability of dying, given you are a Red Shirt?
2. What is the percentage of crew members that have red shirts and died?
3. What is the percentage of blue shirts who survived?
4. What is the probability of dying Given you are a Gold Shirt?

BIVARIATE CATEGORICAL DATA

- Next we may find **conditional relative frequencies**

	😃	☠️	
🔵	0.9485294	0.0514706	1
🟡	0.8363636	0.1636364	1
🔴	0.8995816	0.1004184	1



BIVARIATE CATEGORICAL DATA

Distribution of
Conditional Relative Frequencies

	😃	☠️	
🔵	0.9485294	0.0514706	1
🟡	0.8363636	0.1636364	1
🔴	0.8995816	0.1004184	1

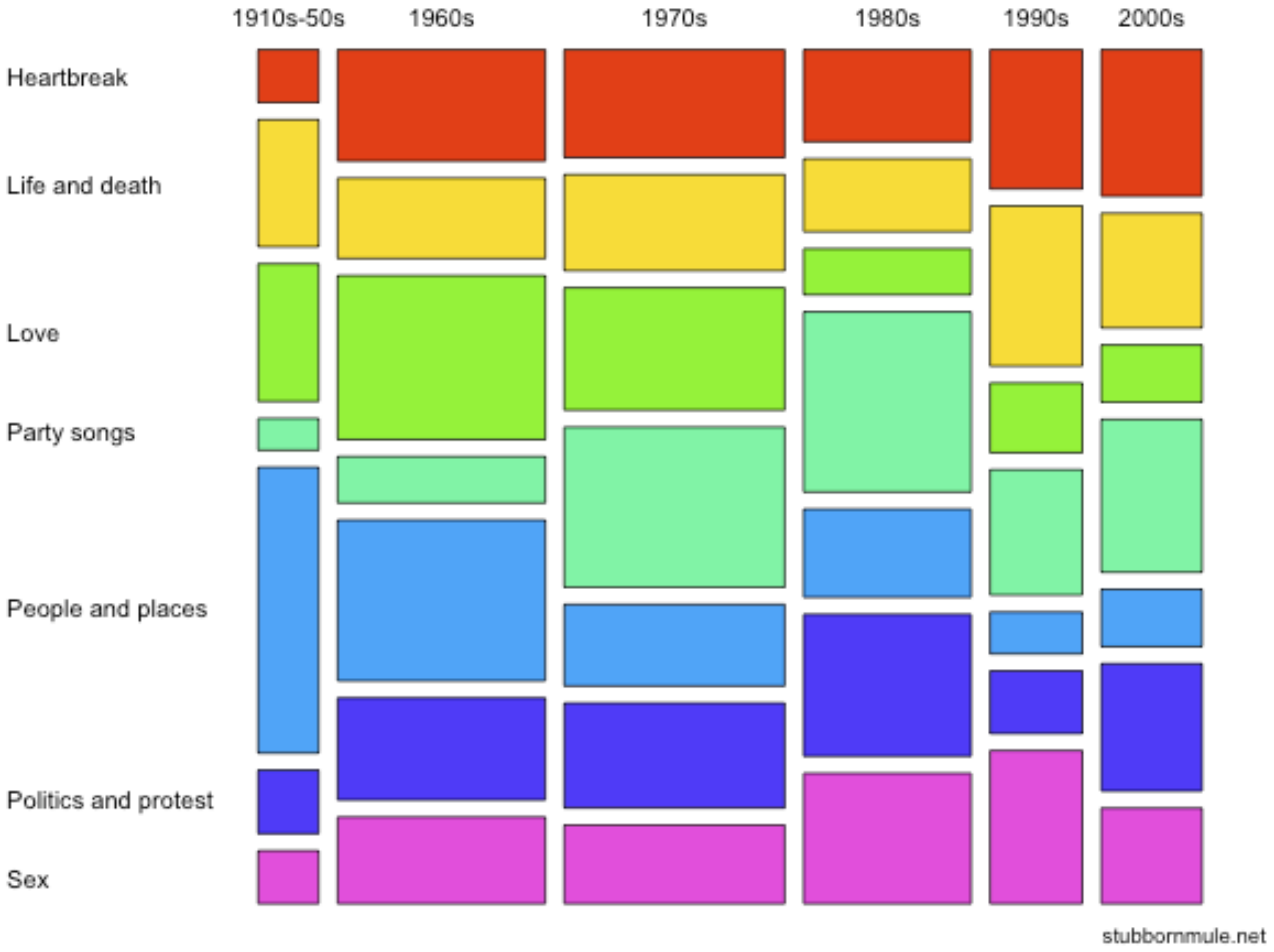
If shirt colour is **Independent** of Status, then the probability of dying should be the same regardless of shirt colour.

Chi-Square Tests (For Later)

BIVARIATE CATEGORICAL DATA

- Another type of graph used is **Mosaic Plots.** Widths describe how many observations fall in each category.

- Mosaic plot showing cross-sectional distribution through time of different musical themes in the Guardian's list of "1000 songs to hear before you die"



Examples:

- Question 1, Page 107
- Question 2, Page 108
- Question 3, Page 112

Homework: Read Pages 97-104 Barron's, Quiz 6, Quiz 7

BIVARIATE QUANTITATIVE DATA

- We represent relationships between two numeric variables (sample data) using a **scatter plot**.

- When describing a relationship there are several things we must consider
  - **Form**
  - **Direction**
  - **Strength**

EXAMPLE

Is there a relationship between the amount of sugar (in grams) and the number of calories in movie-theatre candy? Here are the data from a sample of 12 types of candy.

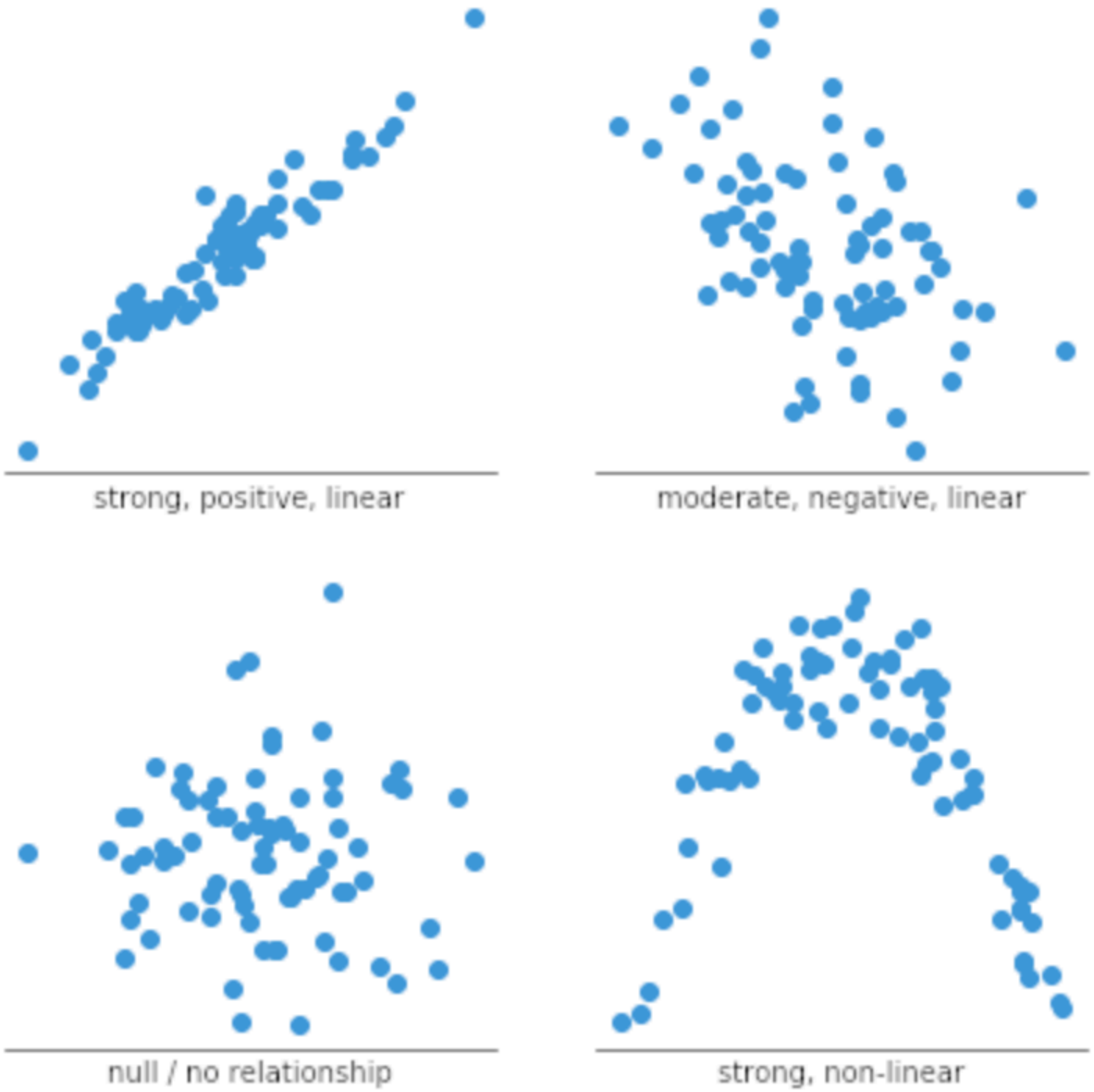
Name	Sugar (g)	Calories
Butterfinger Minis	45	450
Junior Mints	107	570
M&M'S	62	480
Milk Duds	44	370
Peanut M&M'S	79	790
Raisinets	60	420
Reese's Pieces	61	580
Skittles	87	450
Sour Patch Kids	92	490
SweeTarts	136	680
Twizzlers	59	460
Whoppers	48	350

Using your TI-84, plot the data.

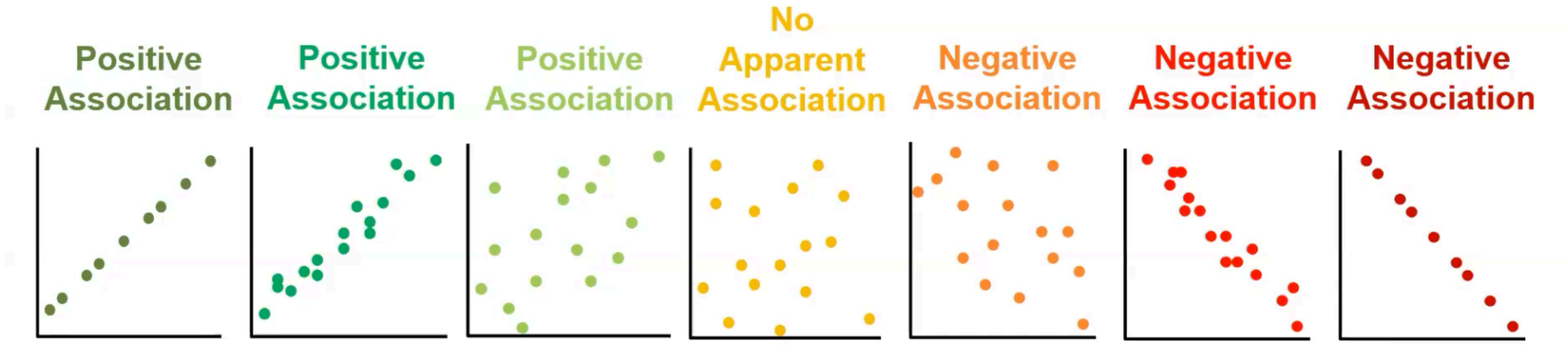
How Would You Describe the Relationship?

BIVARIATE QUANTITATIVE DATA

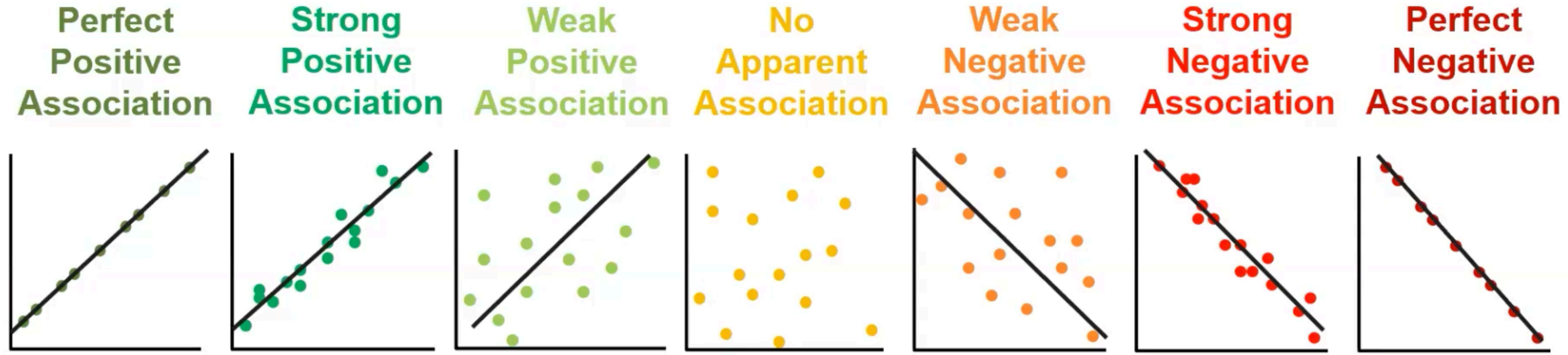
Form of relationship



Direction of relationship



Strength of relationship



Influential Points of relationship

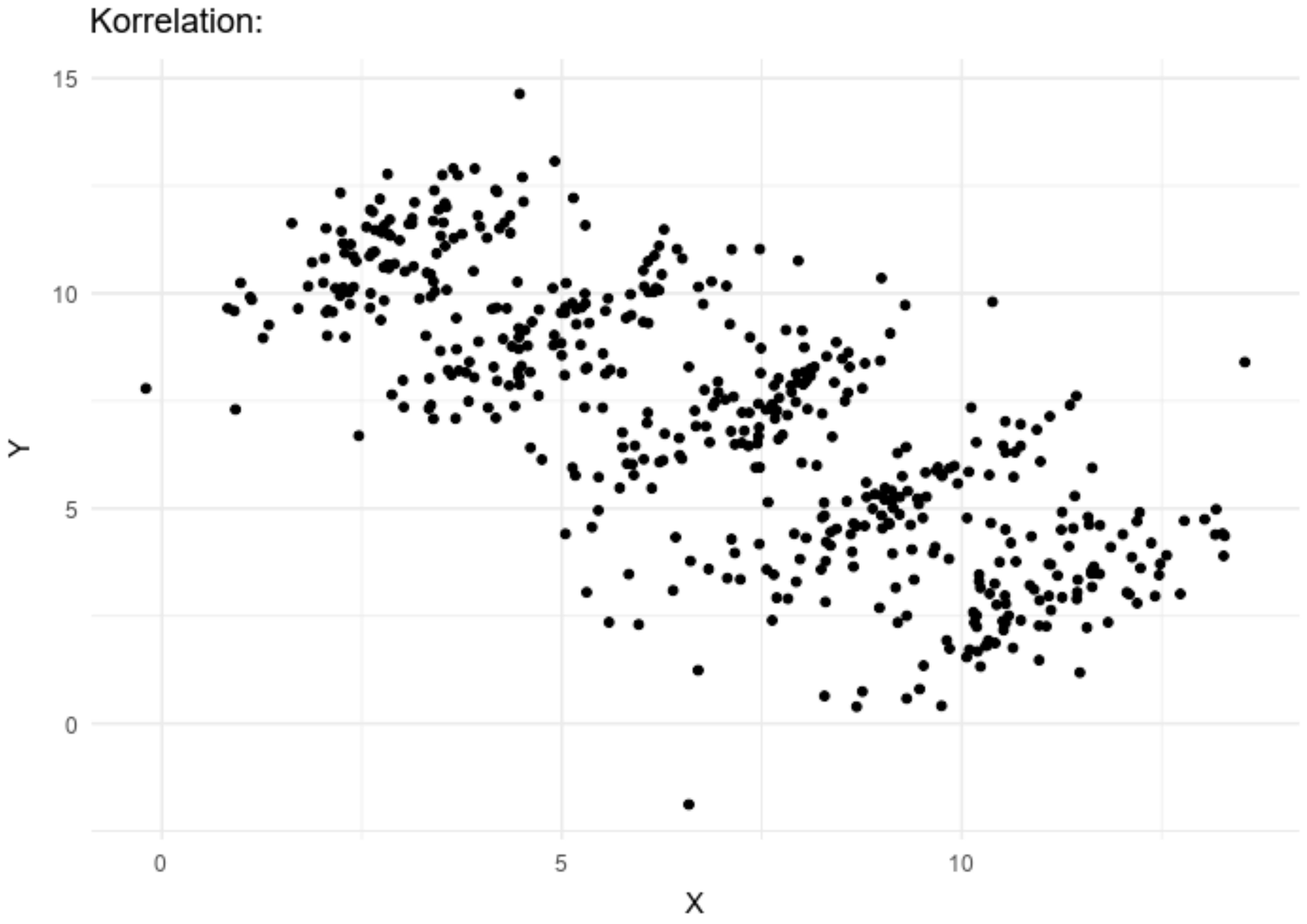
Outliers

Points of High Leverage

https://www.desmos.com/calculator/jwquvmikhr

BIVARIATE QUANTITATIVE DATA

- **Simpson's Paradox:** There is an association within groups of data but the trend disappears, or reverses when groups are combined.
- **Example:** Q7 P.111

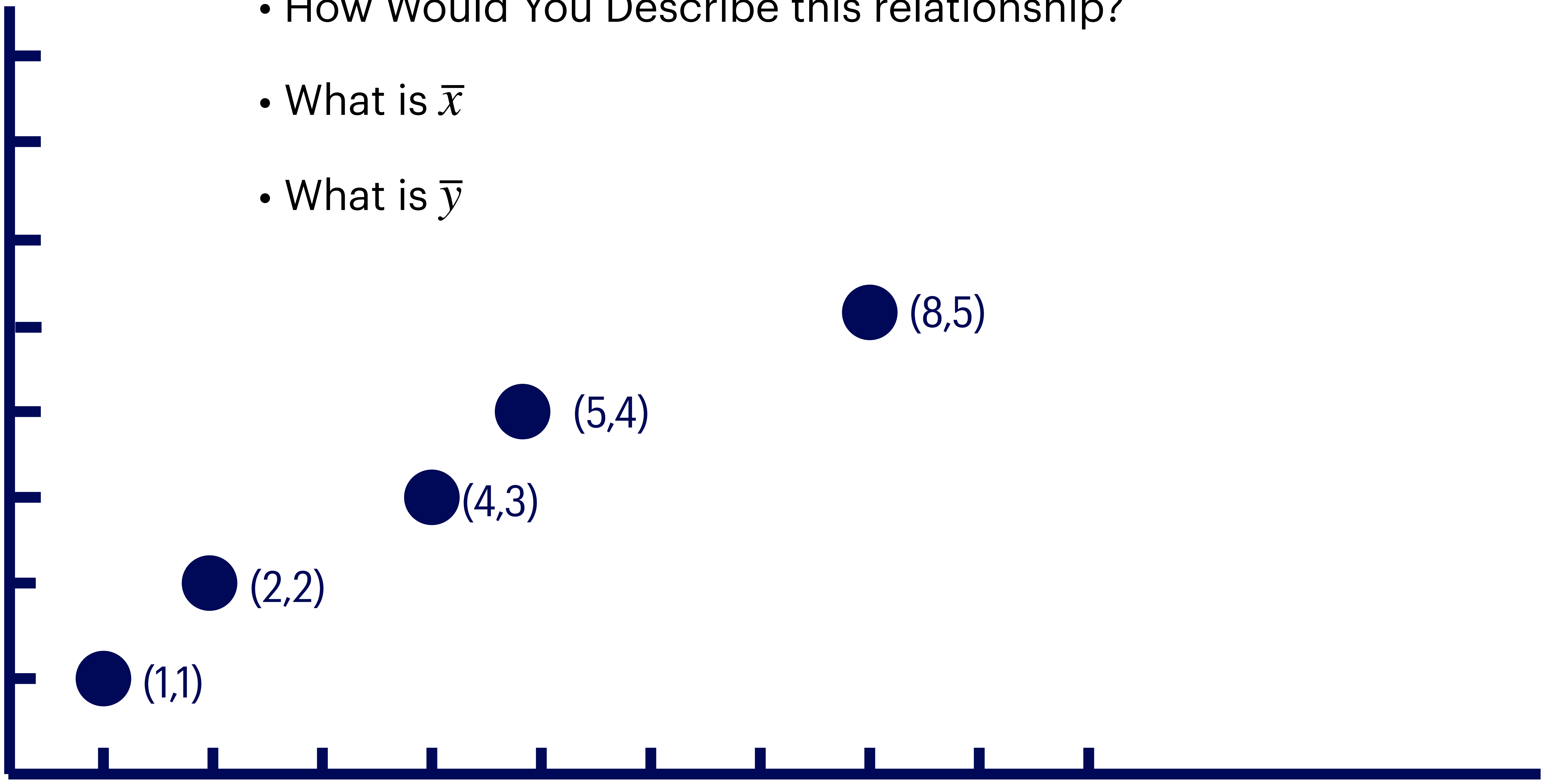


What is the Sample Covariance?

$$Cov(X, Y) = \frac{\sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y})}{n - 1}$$

What is the Sample Covariance?

- How Would You Describe this relationship?
- What is $\bar{x}$
- What is $\bar{y}$



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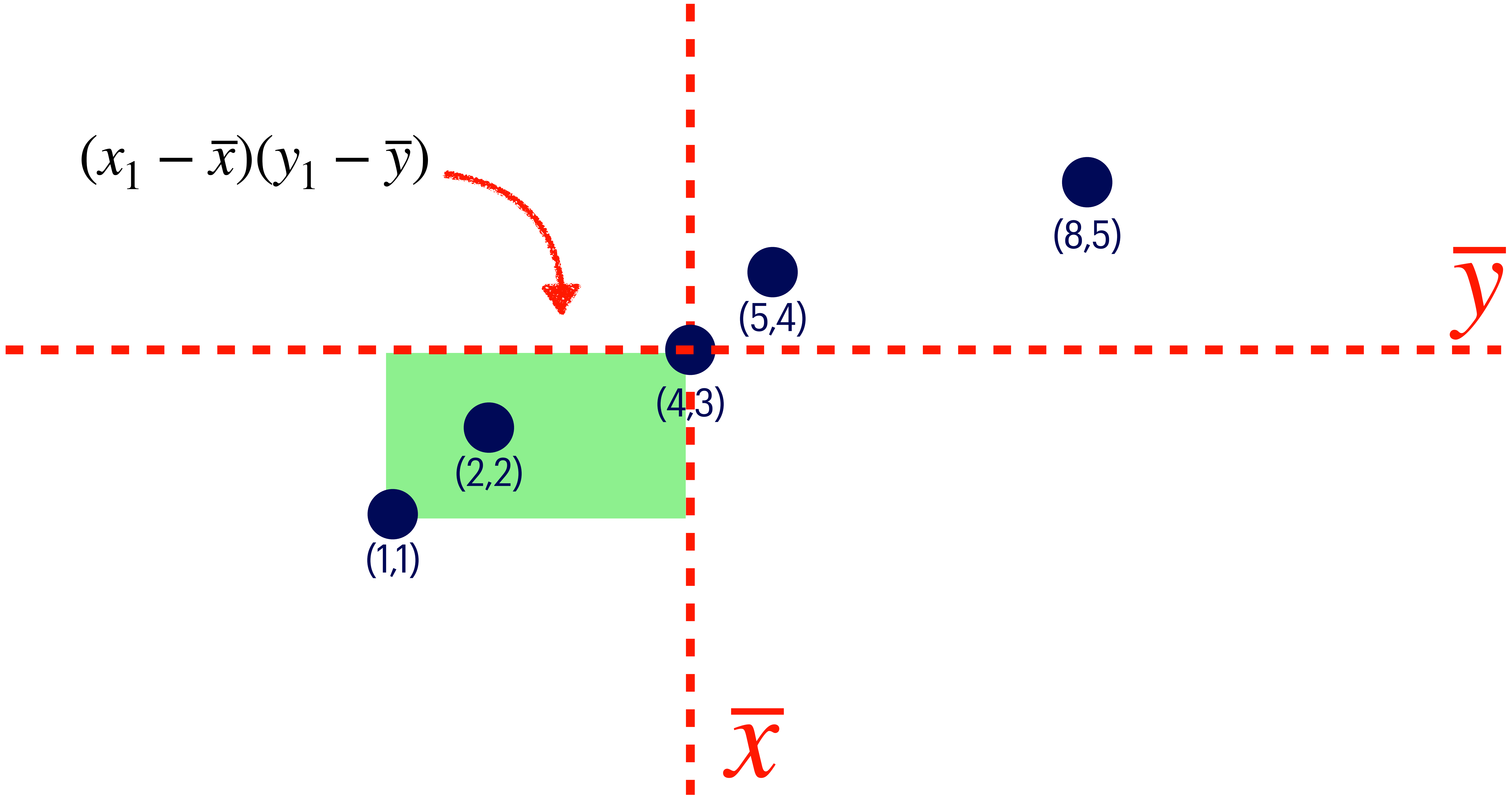

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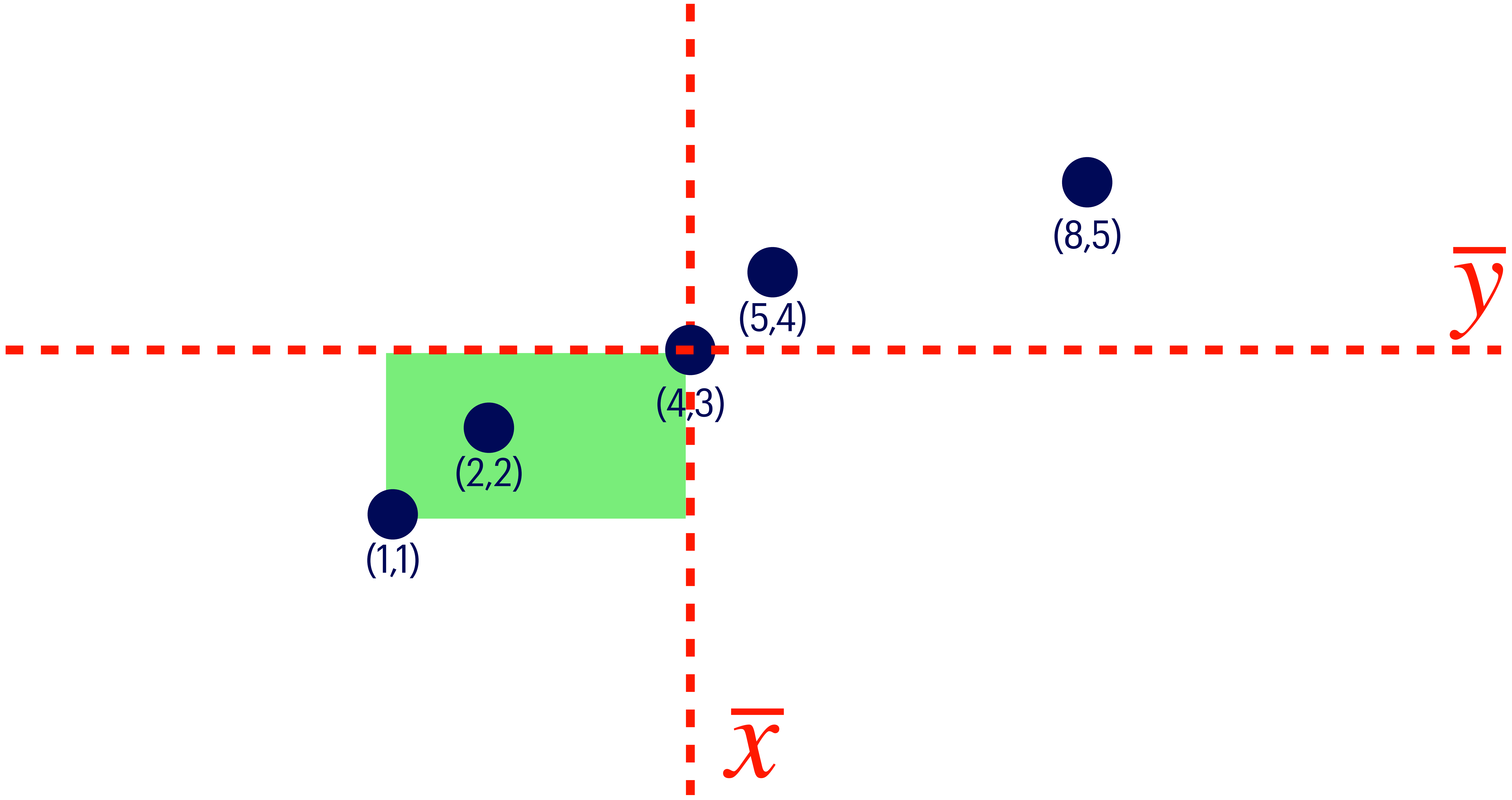
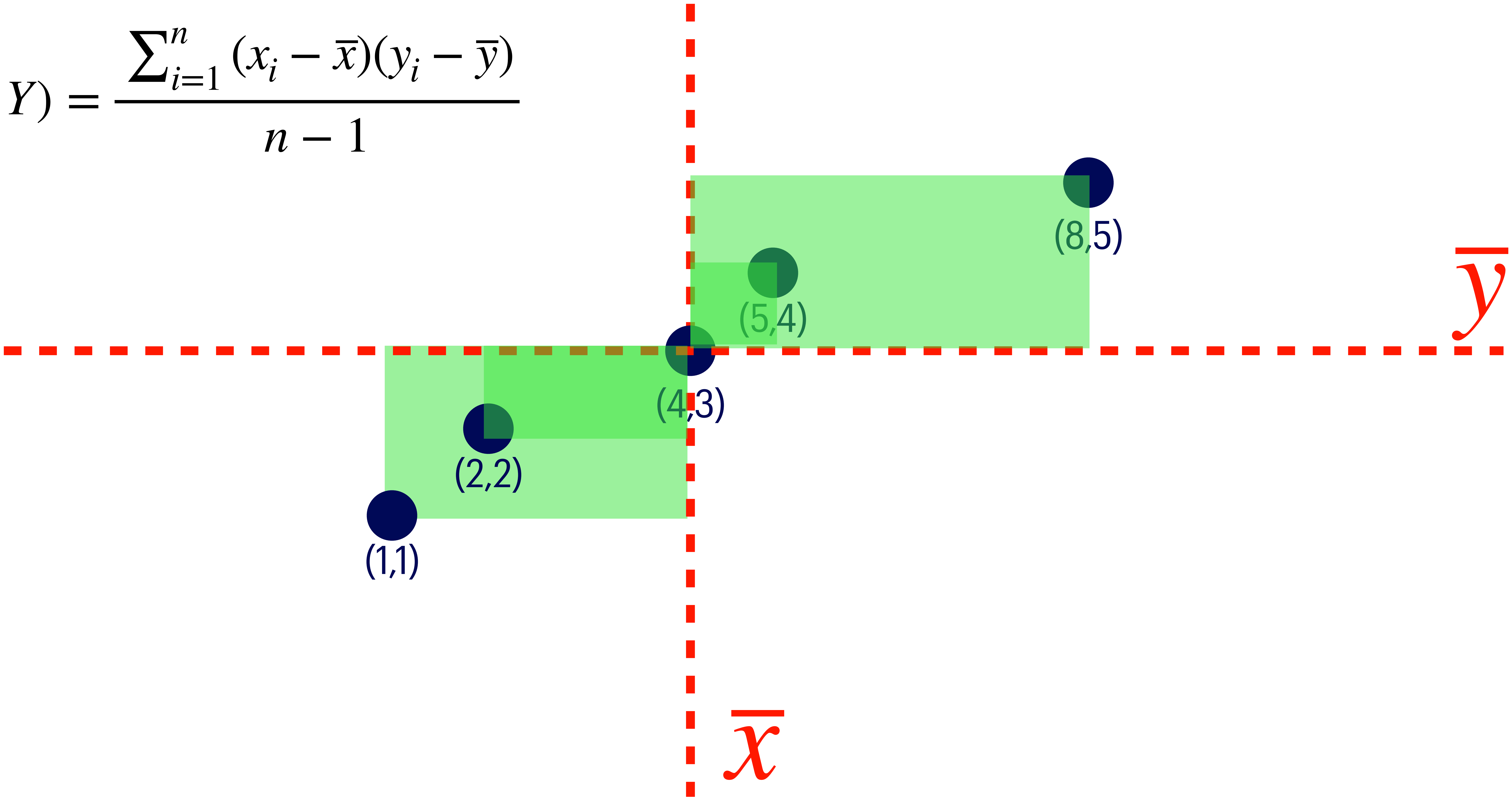


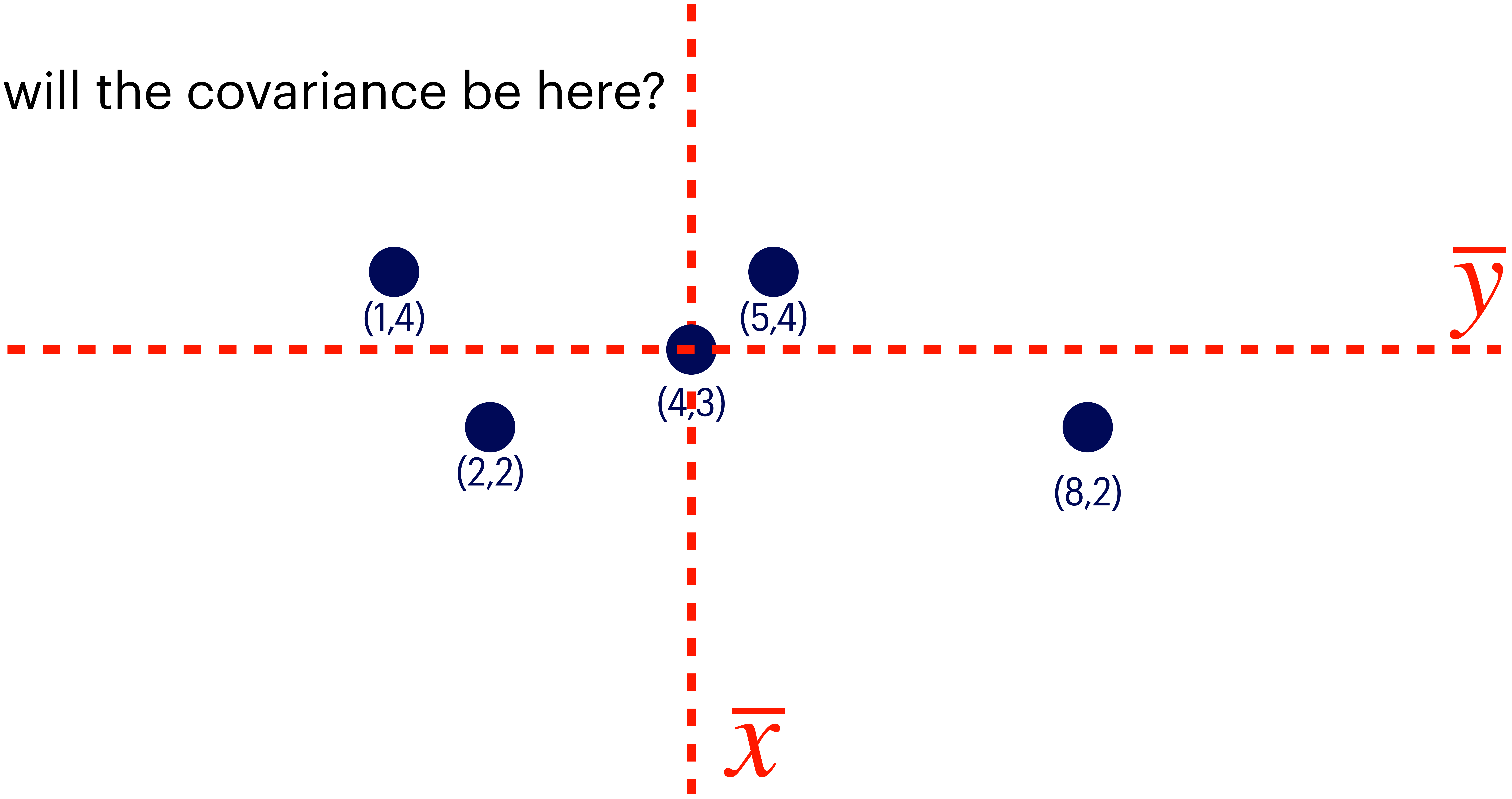
What is the Sample Covariance?

$$Cov(X,Y) = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

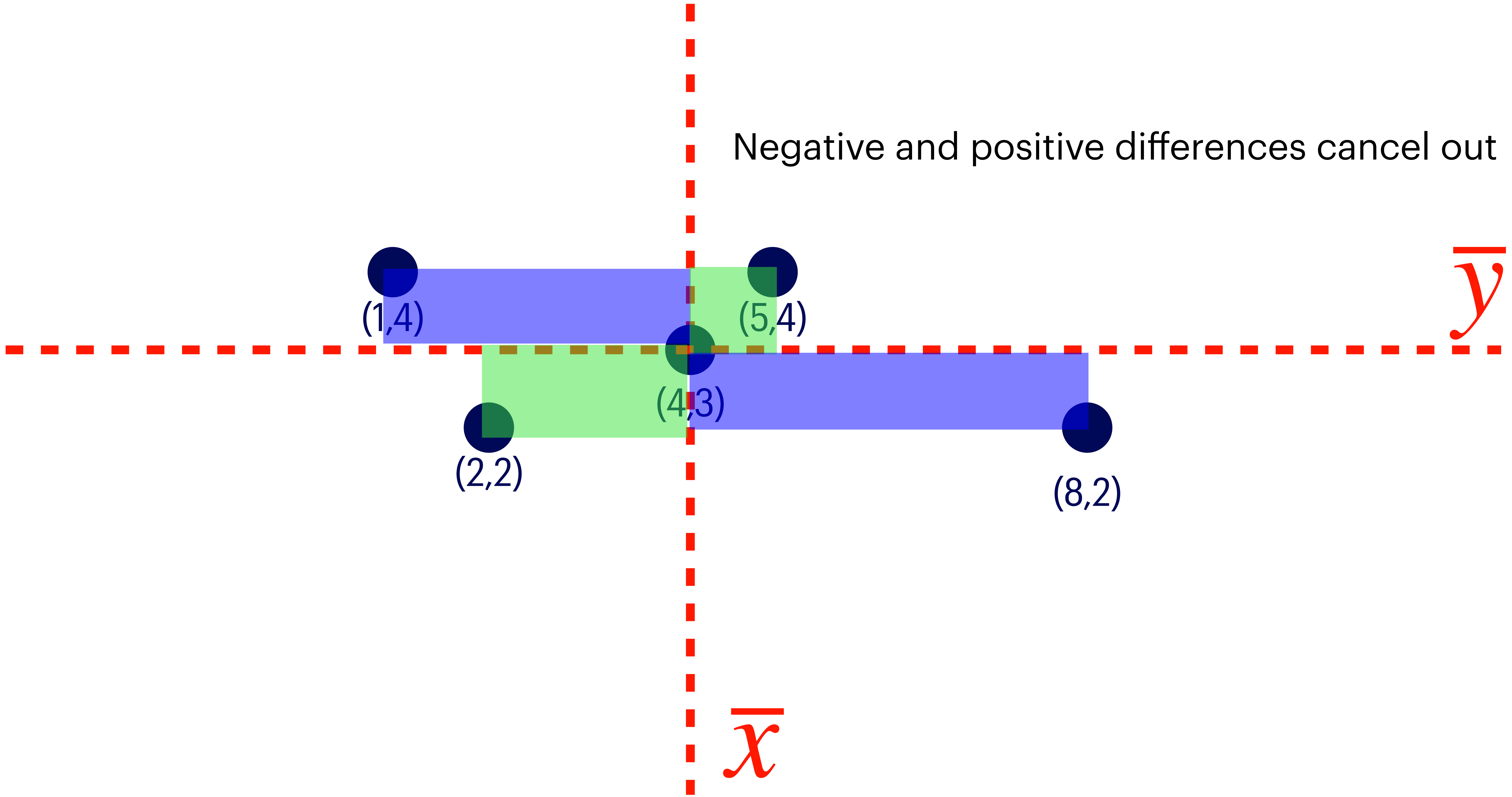


What is the Sample Covariance?

- What will the covariance be here?



What is the Sample Covariance?



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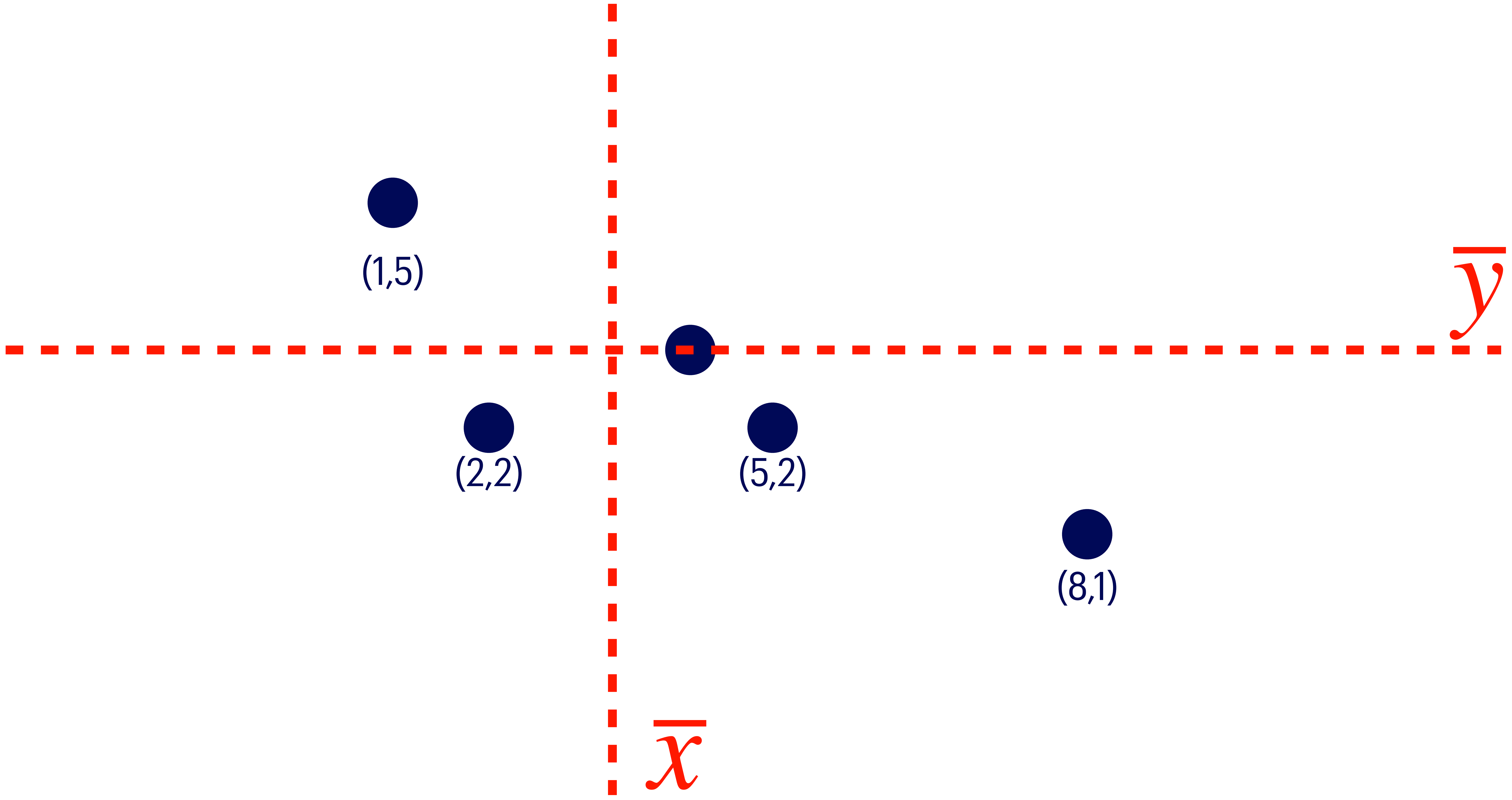
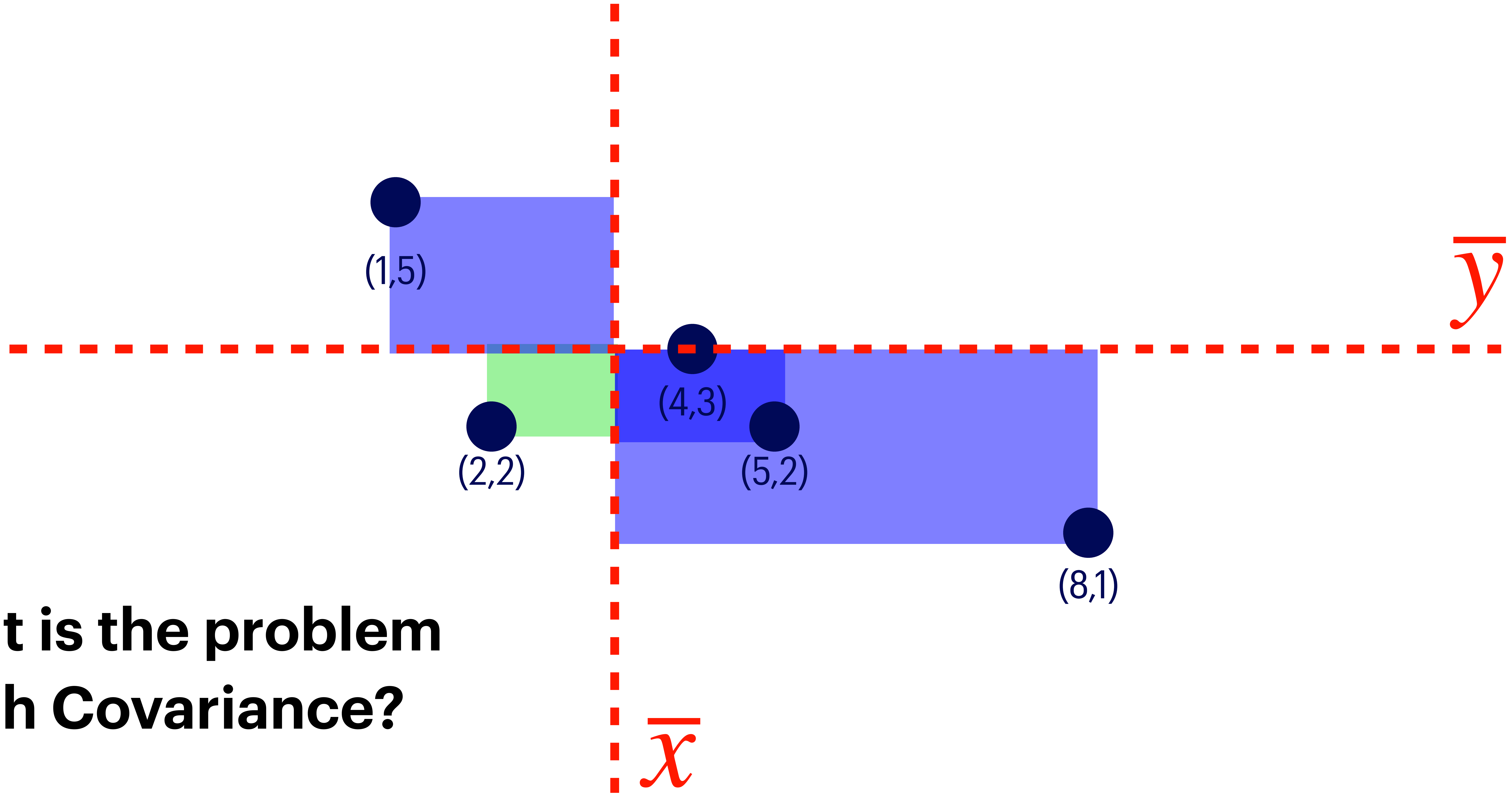


What is the Sample Covariance?



What is the problem with Covariance?

More Examples: http://digitalfirst.bfwpub.com/stats_applet/stats_applet_5_correg.html

What is the Sample Correlation?

$$Cov(X, Y) = \frac{\sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y})}{n - 1}$$

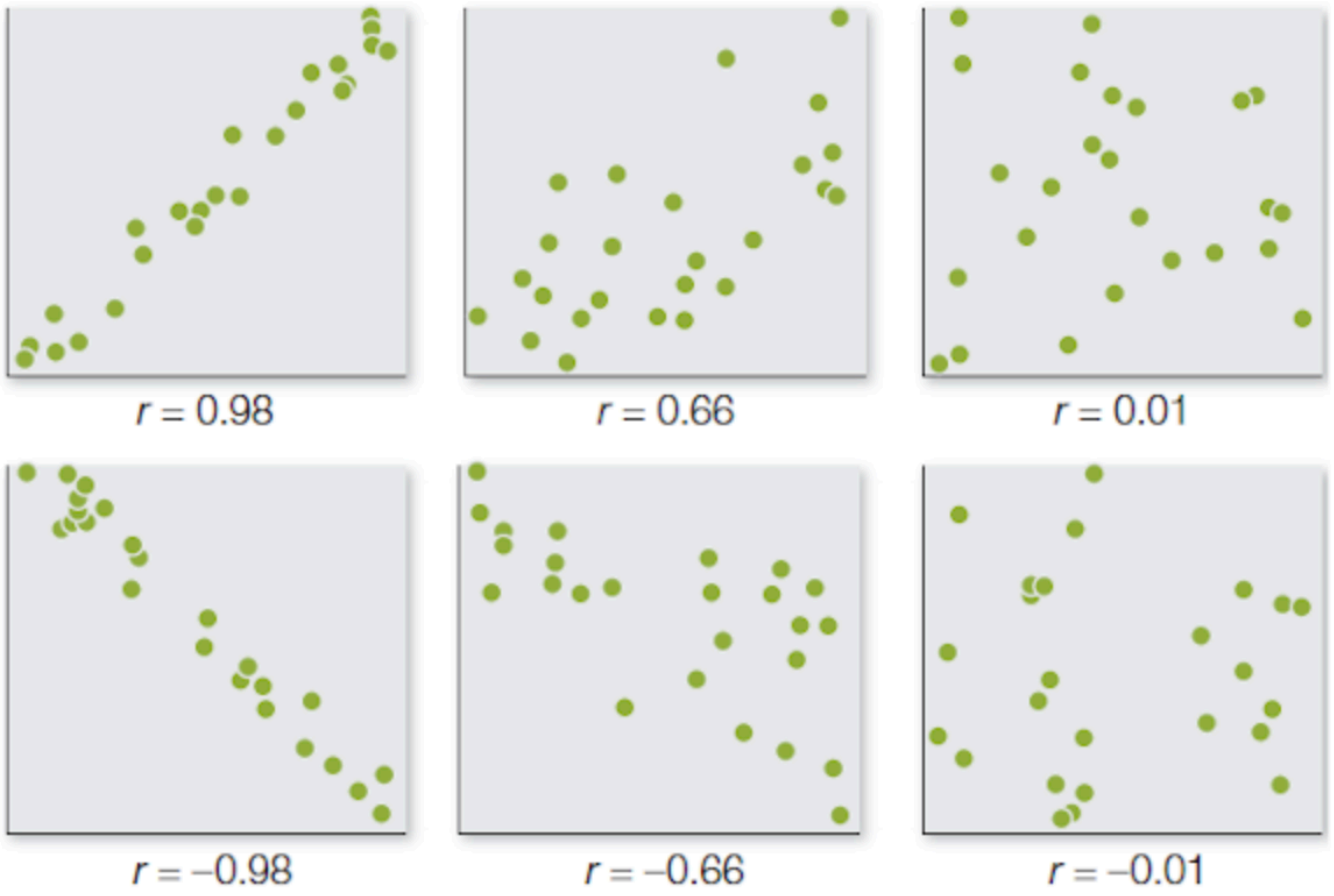
$$r = \frac{Cov(X, Y)}{s_x s_y} = \frac{1}{n - 1} \sum_{i=1}^{n} \left(\frac{x_i - \overline{x}}{s_x} \right) \left(\frac{y_i - \overline{y}}{s_y} \right)$$

Note that *r* is
a **statistic**

What Does Correlation Measure?

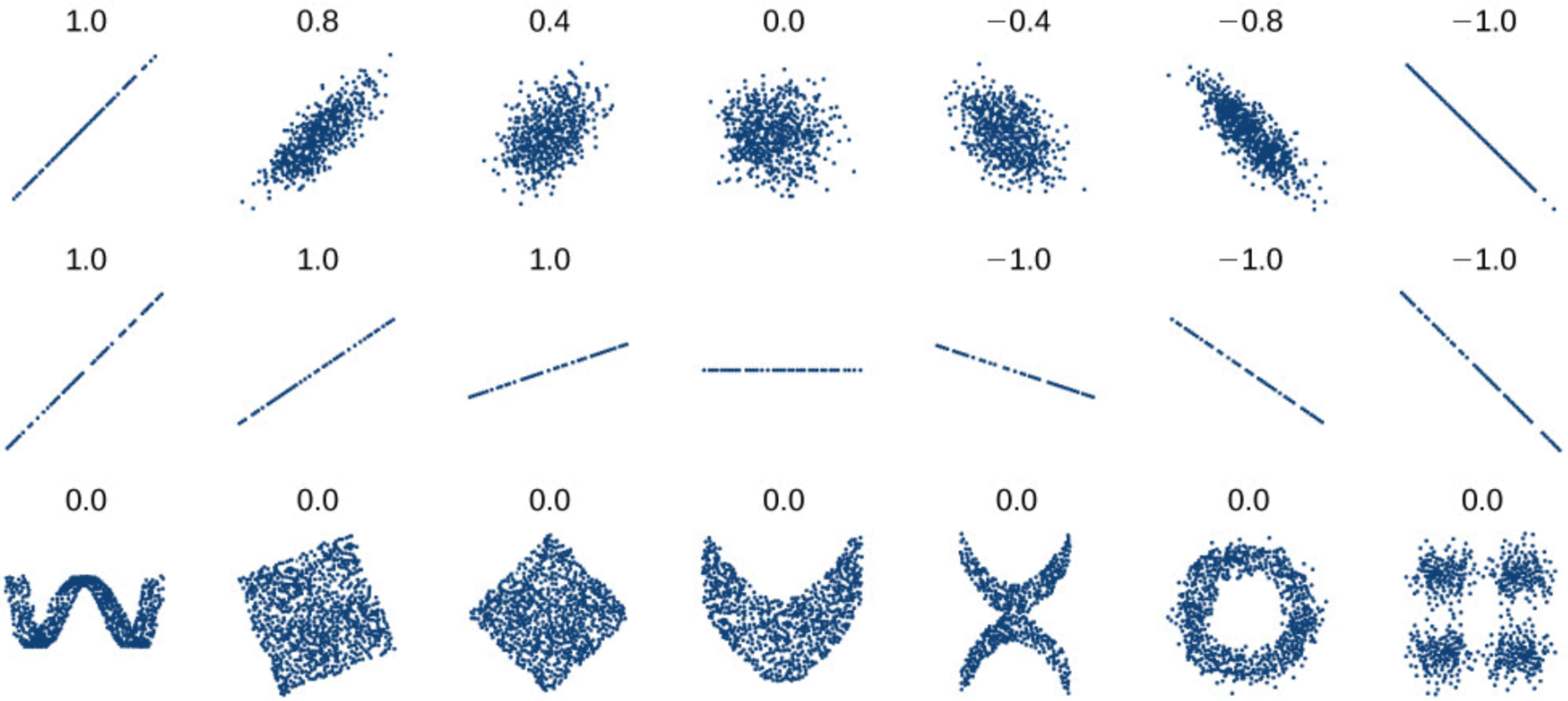
- Direction
- Strength

Guess the Correlation https://www.rossmanchance.com/applets/2021/guesscorrelation/GuessCorrelation.html



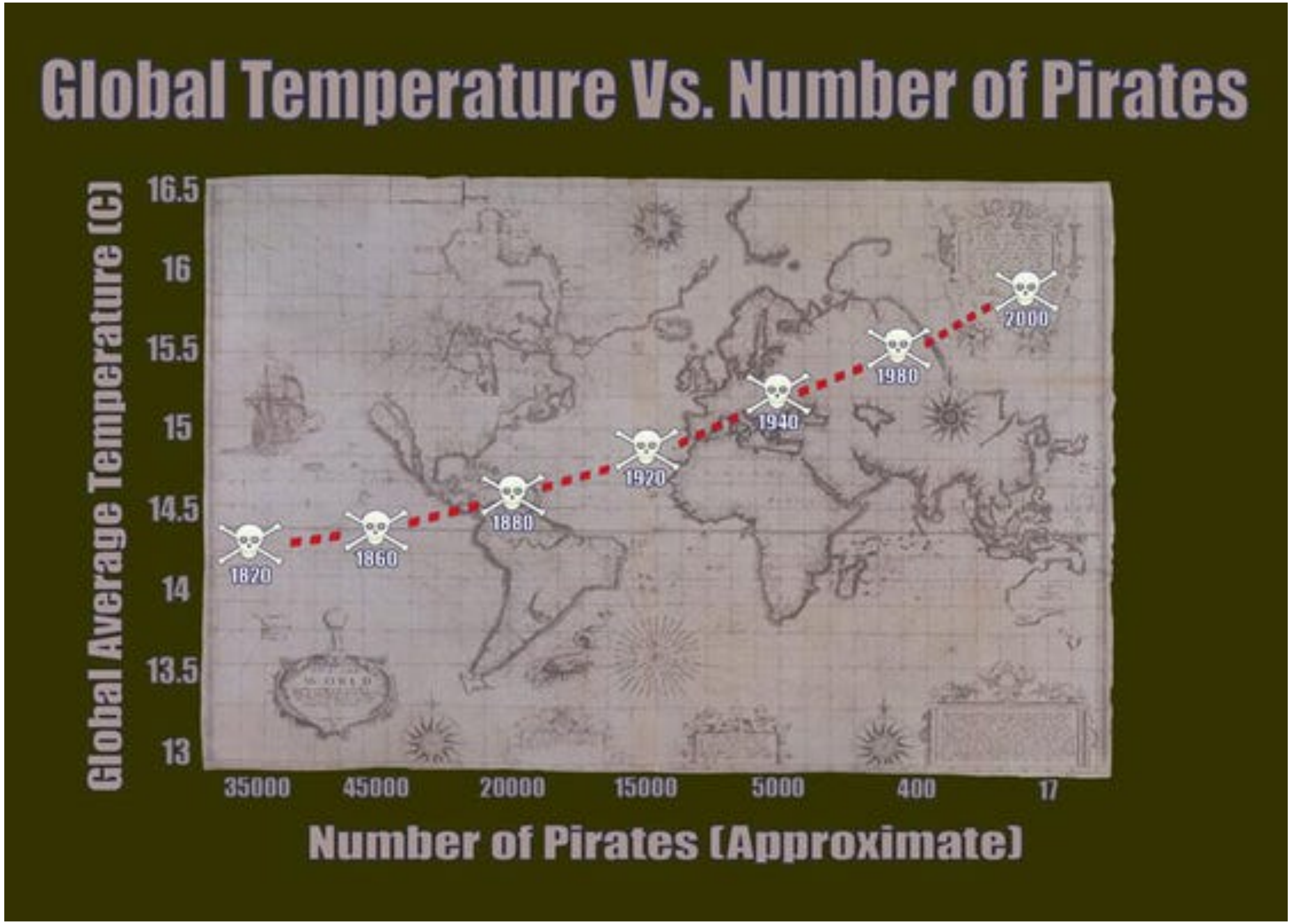
BIVARIATE QUANTITATIVE DATA

ALWAYS PLOT THE DATA

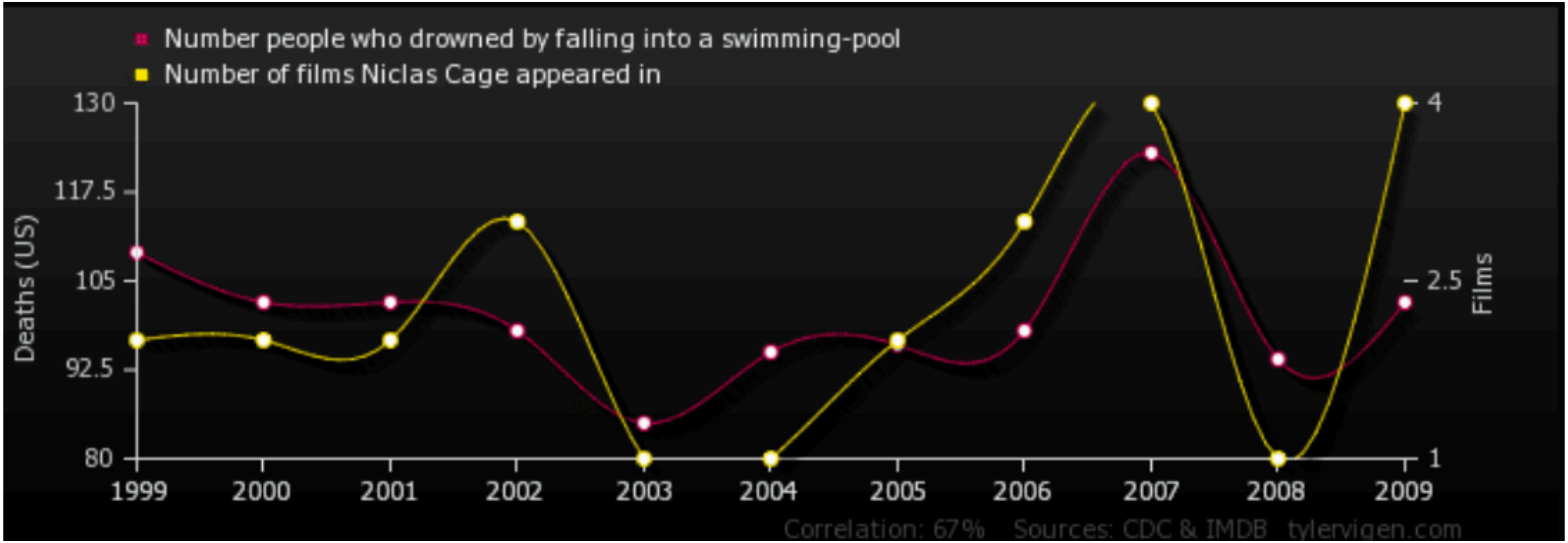


What does correlation tell us about causation?

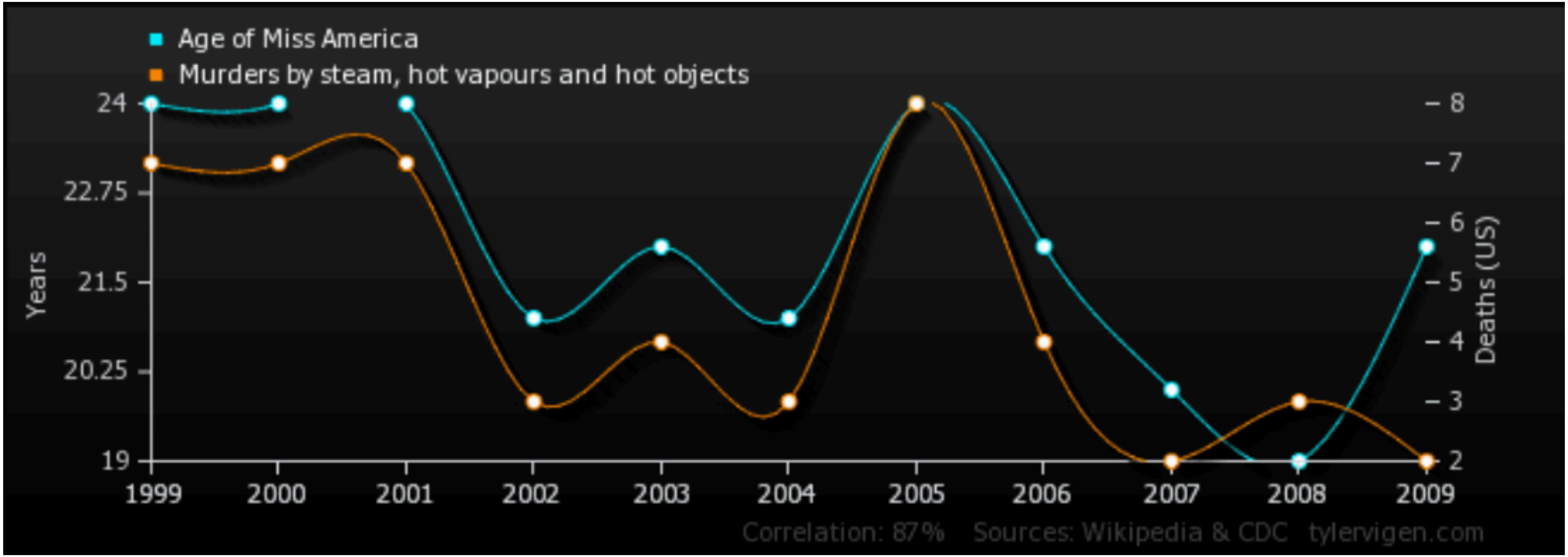
- Is a lack of pirates causing global warming?
- Are Ice Cream Salesman responsible for increased drowning fatalities?



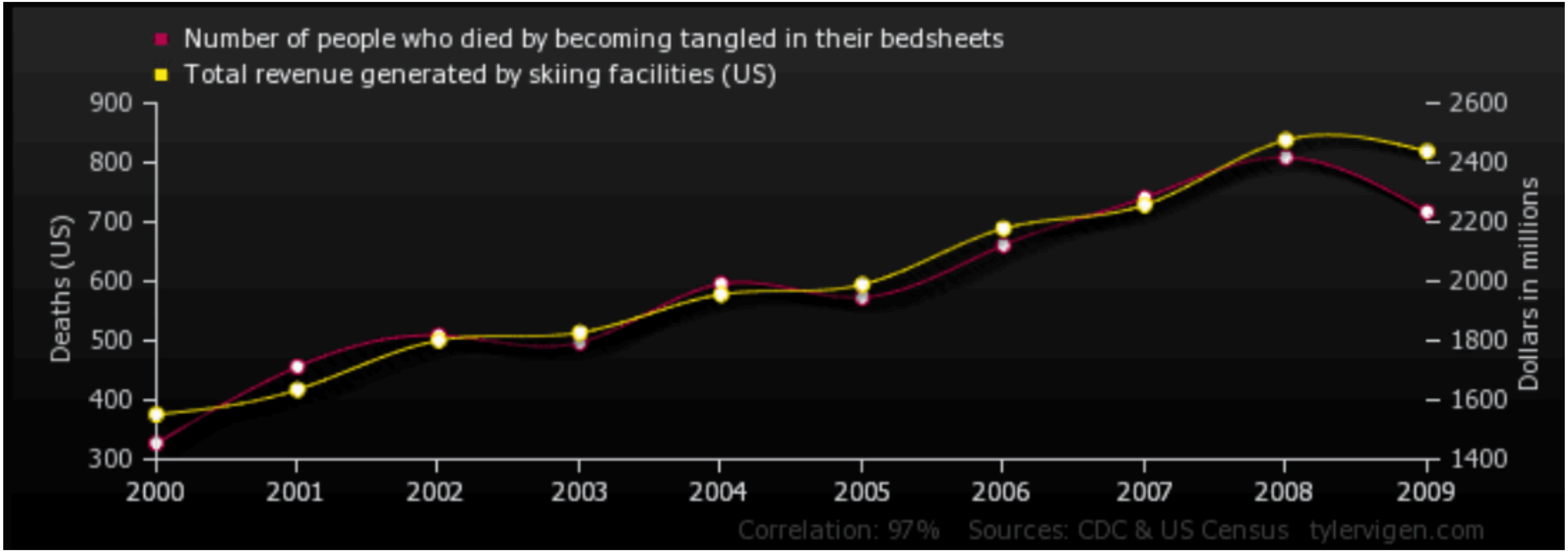
Correlation ≠ Causation Examples



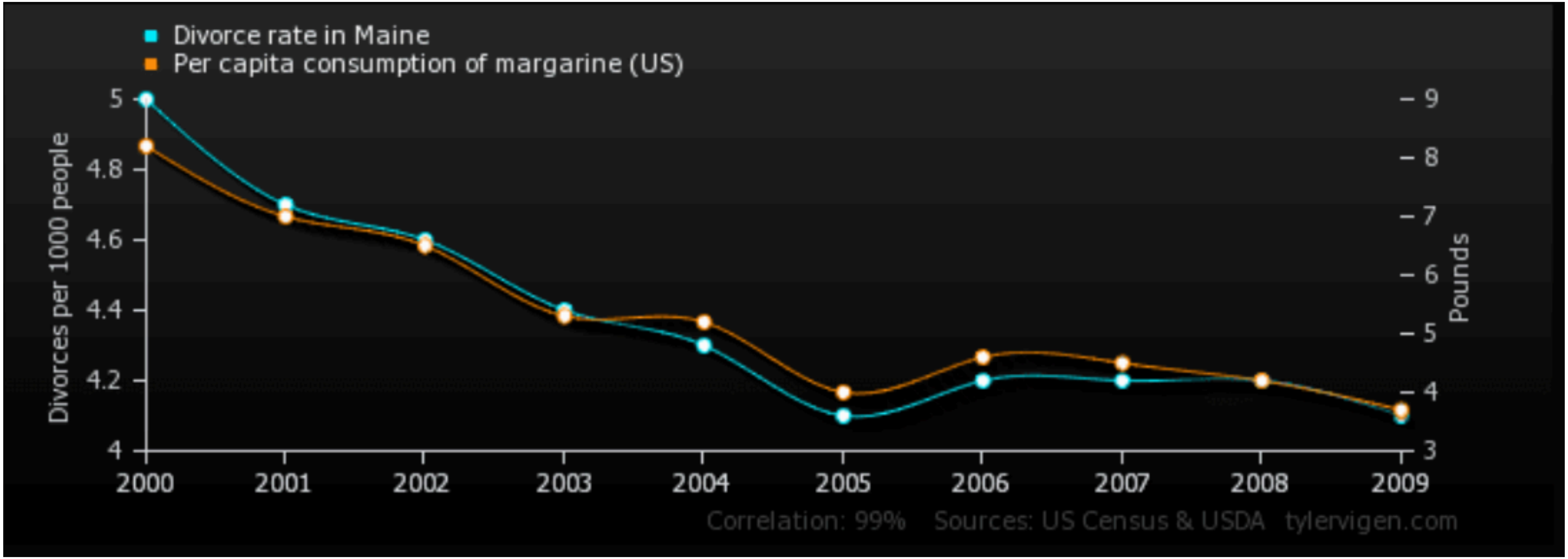
Correlation ≠ Causation Examples



Correlation ≠ Causation Examples

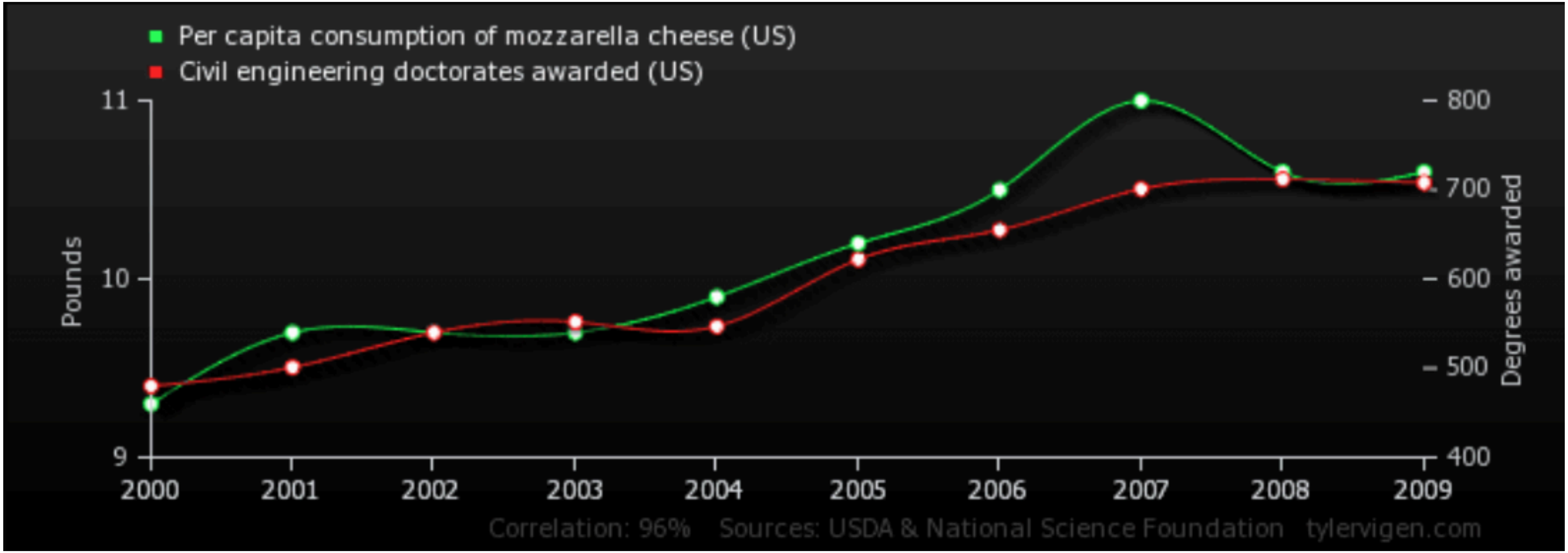


Correlation $\neq$ Causation Examples



Correlation $\neq$ Causation Examples

Correlation $\neq$ Causation Examples



How do we “prove” something is a causal relationship?

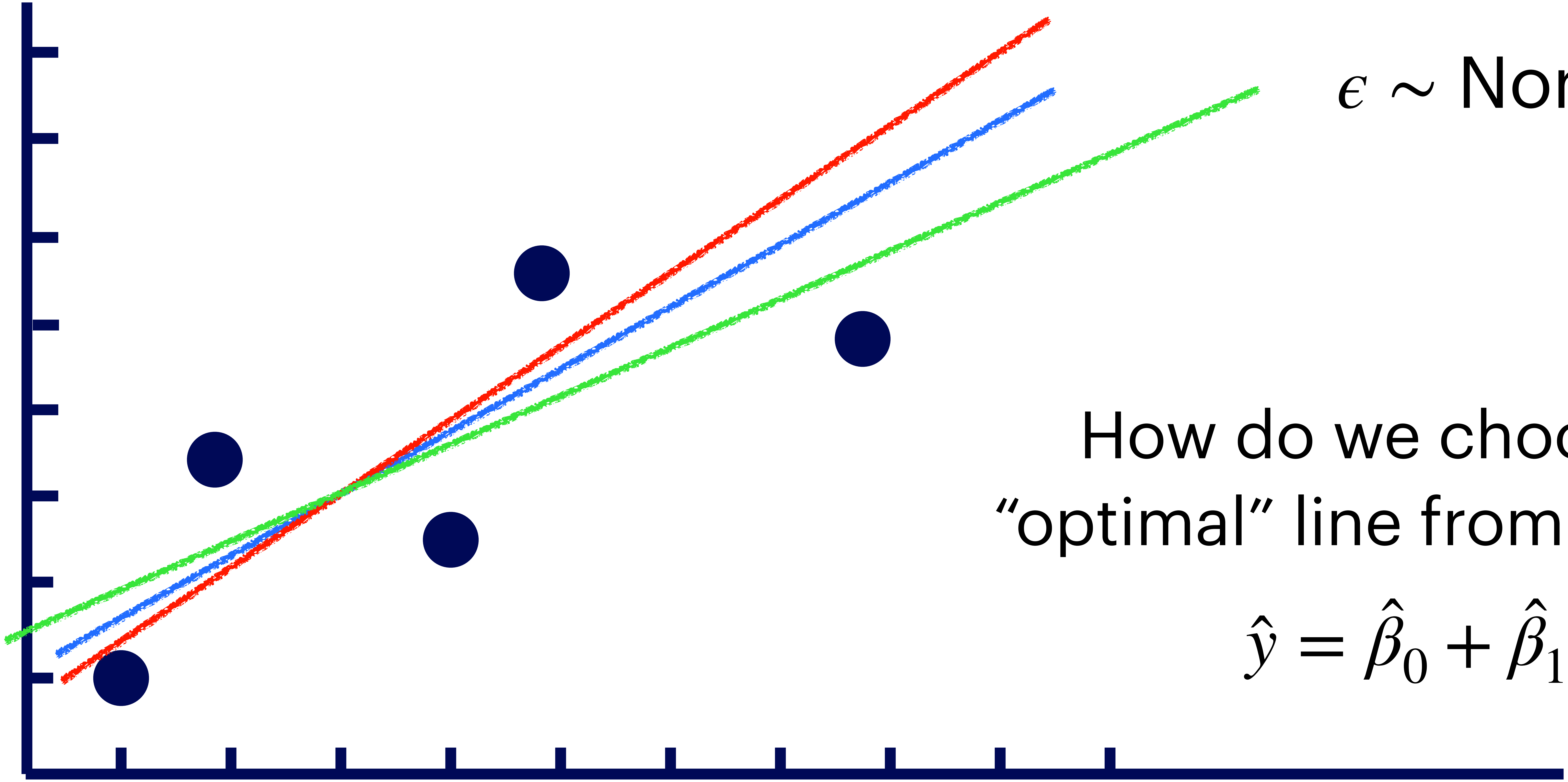
Experiments will be discussed in more detail later (unit 3)

BIVARIATE QUANTITATIVE DATA

Simple Linear Regression Model

$$y = \beta + \beta_1 x + \epsilon$$

$$\epsilon \sim \text{Normal}(0,\sigma^2)$$

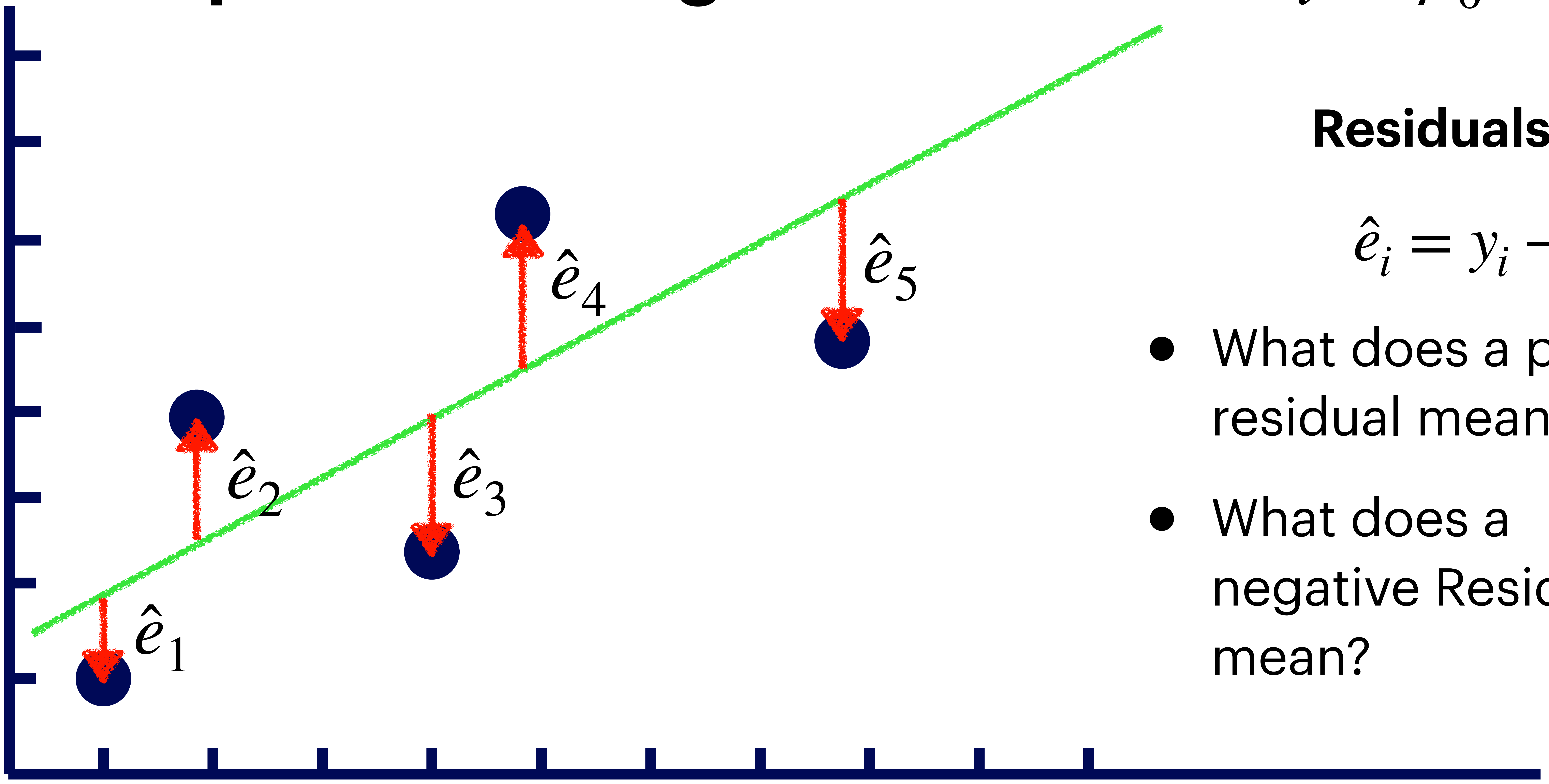


How do we choose the "optimal" line from the data?

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

Simple Linear Regression Model

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$$

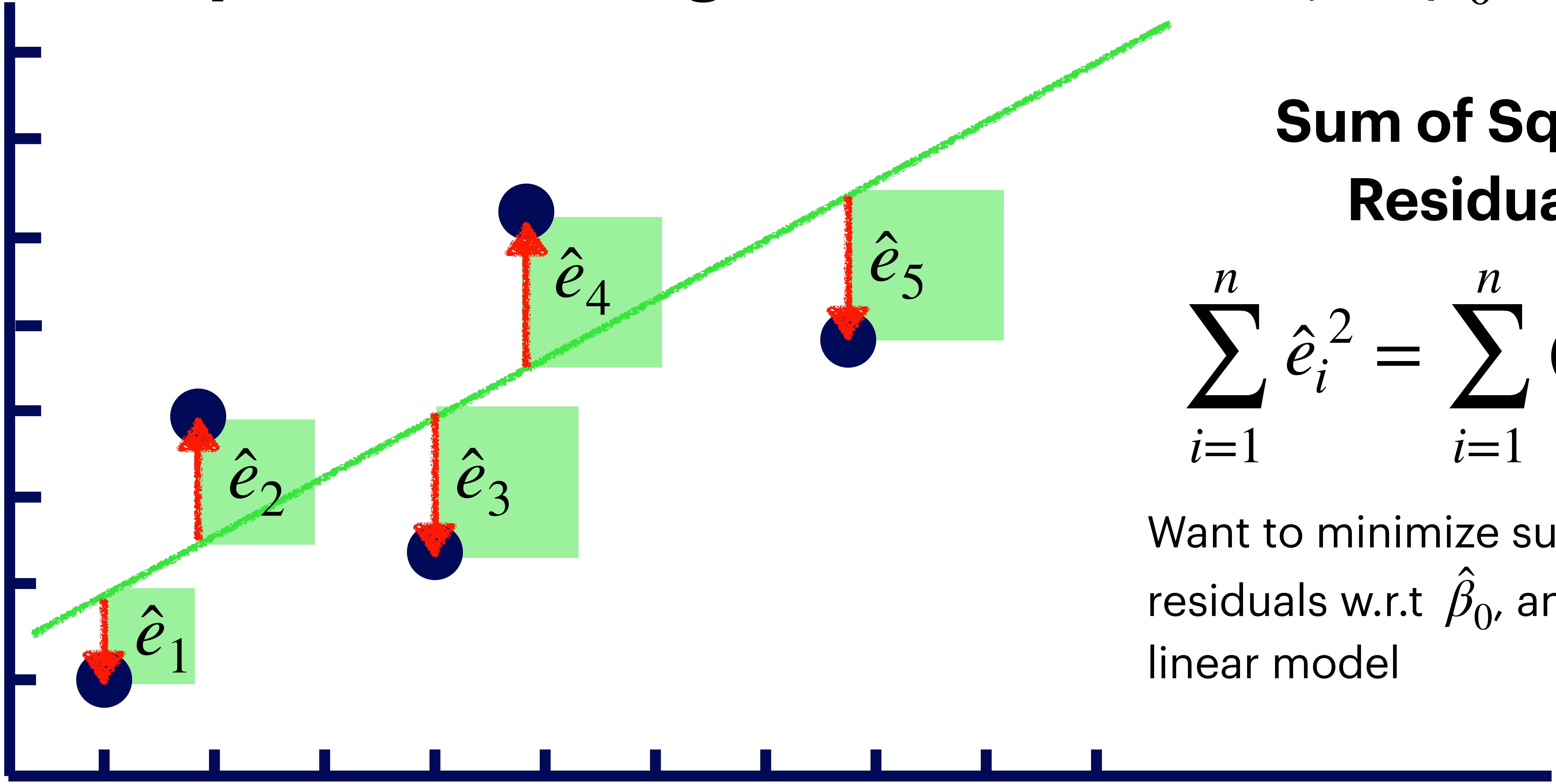


Residuals

$$\hat{e}_i = y_i - \hat{y}_i$$

- What does a positive residual mean?
- What does a negative Residual mean?

Simple Linear Regression Model $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$



Sum of Square Residuals

$$\sum_{i=1}^{n} \hat{e}_i^{\,2} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

Want to minimize sum of square residuals w.r.t $\hat{\beta}_0$, and $\hat{\beta}_1$ to get linear model

The Derivation

$$0 = \frac{\partial}{\partial \hat{\beta}_0} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$0 = \frac{\partial}{\partial \hat{\beta}_1} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

https://www.desmos.com/calculator/lywhybetzt

Formulas

$$\hat{\beta}_1 = r\frac{s_y}{s_x} = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{n}(x_i - \overline{x})^2} \qquad b = r\frac{s_y}{s_x} \qquad b_1 = r\frac{s_y}{s_x}$$

$$\hat{\beta}_0 = \overline{y} - \hat{\beta}_1\overline{x} \qquad a = \overline{y} - b\overline{x} \qquad b_0 = \overline{y} - b_1\overline{x}$$

BIVARIATE QUANTITATIVE DATA

$$\bar{y} = \hat{\beta}_0 + \hat{\beta}_1 \bar{x}$$

Don't forget that our line of best fit will always pass through $(\bar{x}, \bar{y})$

$$\bar{y} = a + b\bar{x}$$

BIVARIATE QUANTITATIVE DATA

$\hat{\beta}_0$ Represents the average value of “y” when “x” is zero. This is often meaningless

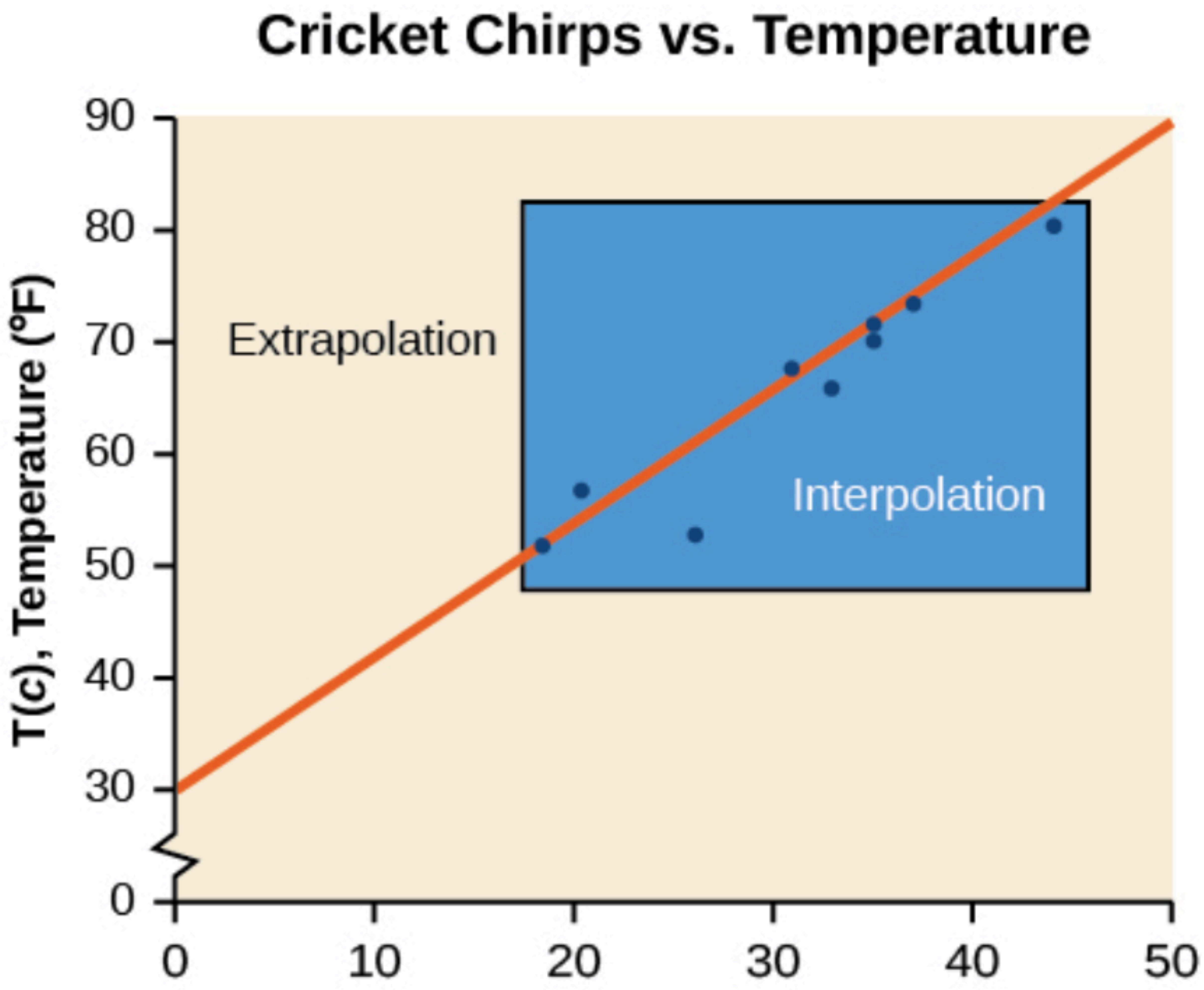
$\hat{\beta}_1$ Represents the average increase in “y” for a **one unit** change in “x”. Think Rise/One

Making Predictions: What does a prediction mean?

Average value of y given value of x. “Using our model we would predict an average temperature of Y for x Cricket Chirps in 15 seconds.

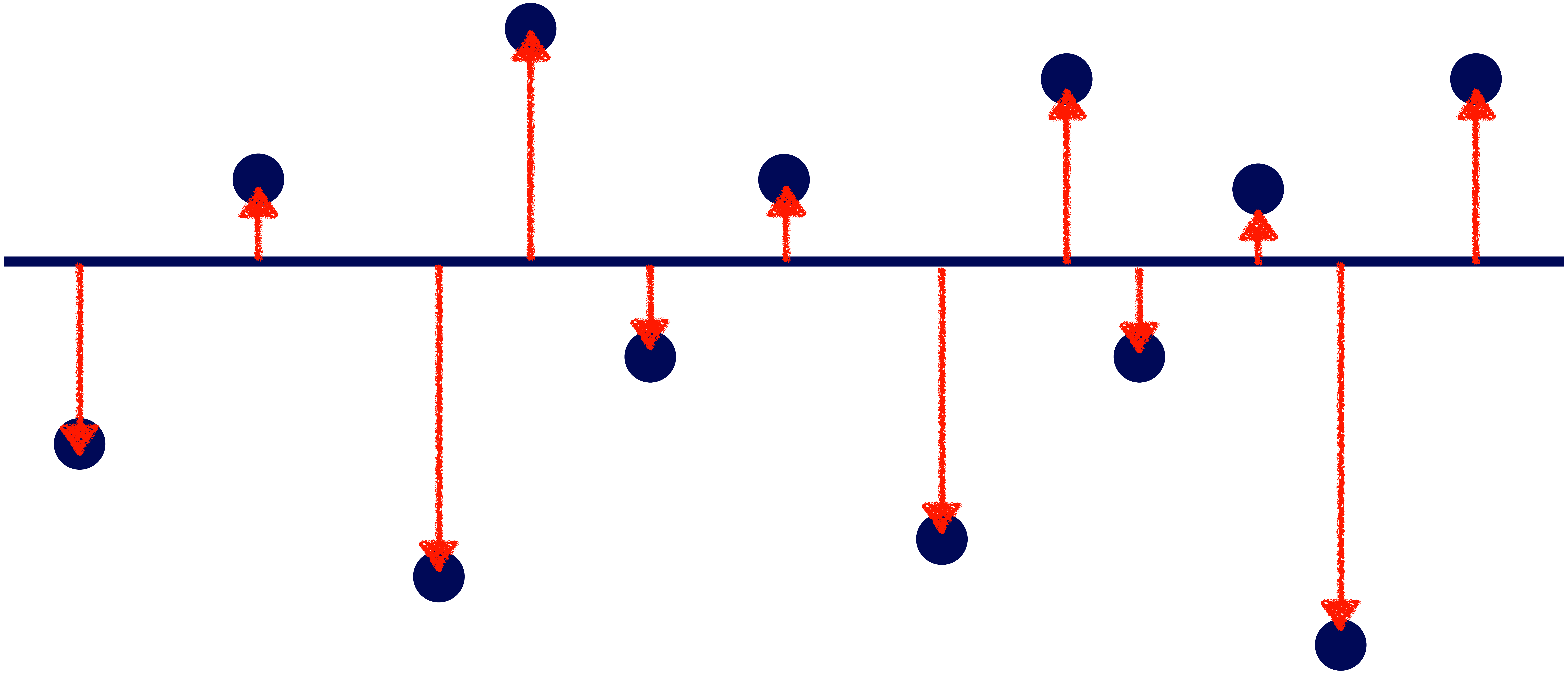
What is extrapolation?

What does 0 cricket chirps in 15 seconds tell us?

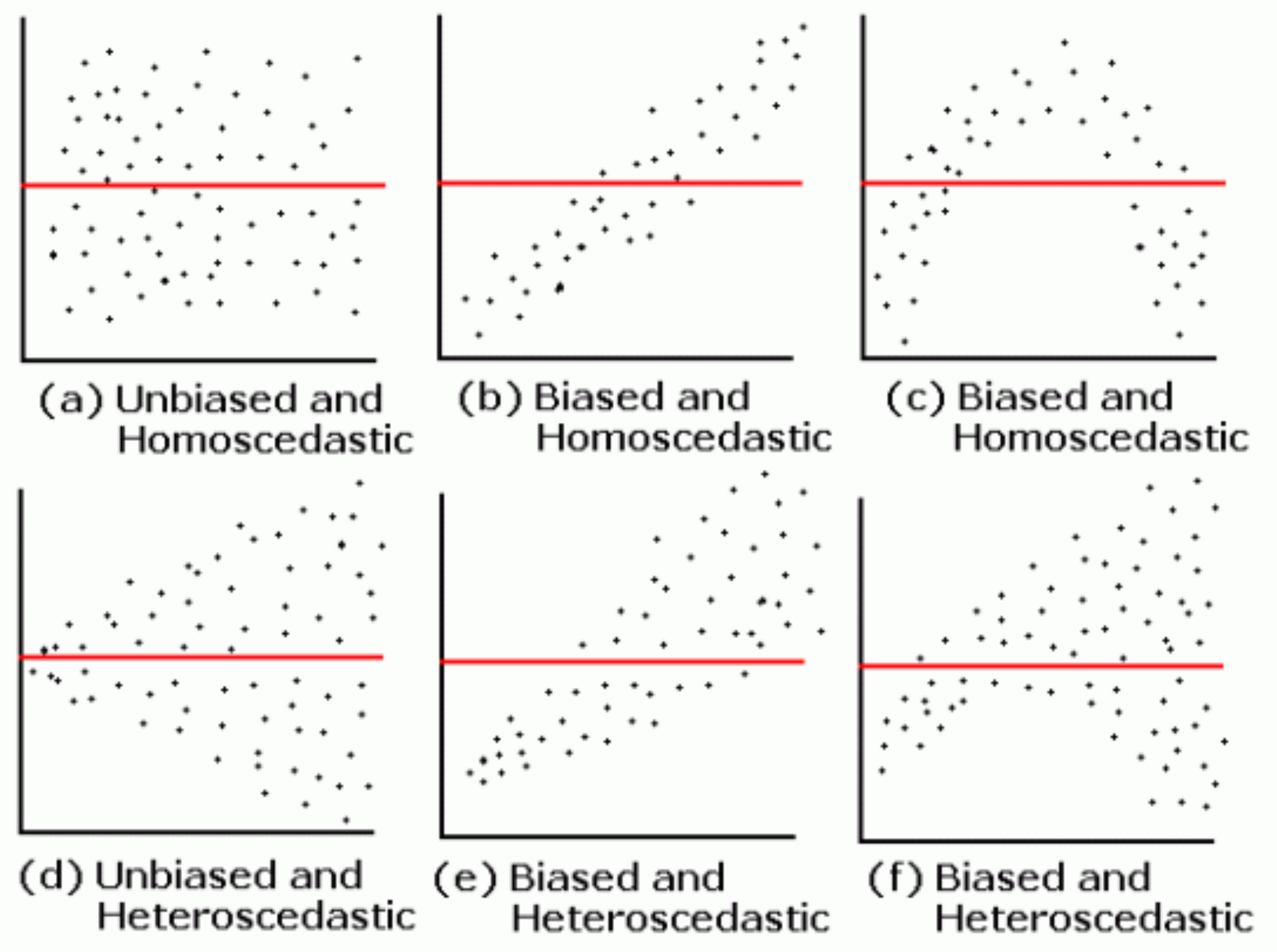


Model Validation: What is a residual plot?

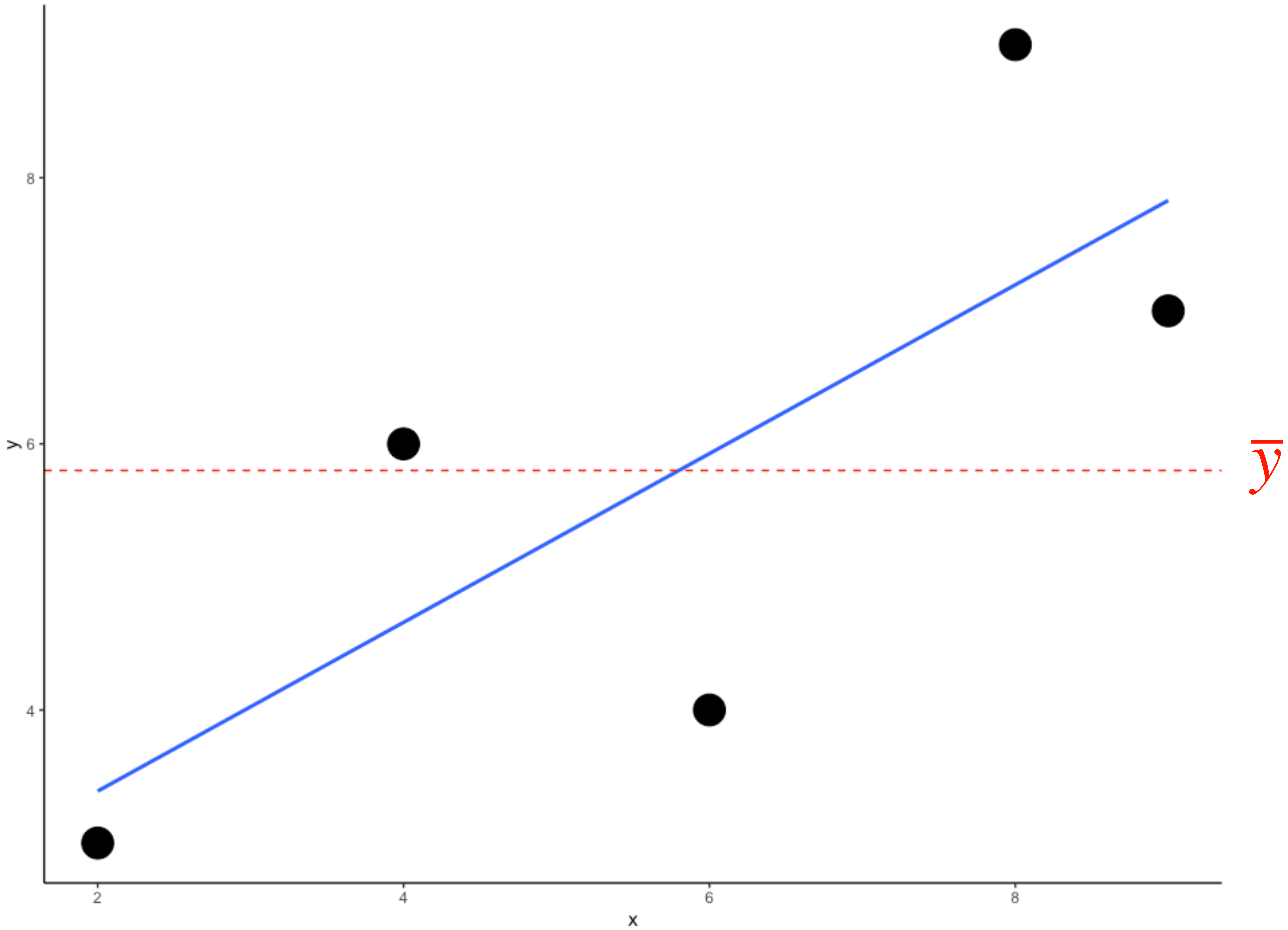
Recall Our Model: $y = \beta + \beta_1 x + \epsilon$ $\epsilon \sim \text{Normal}(0,\sigma^2)$



RESIDUAL PLOTS



BIVARIATE QUANTITATIVE DATA



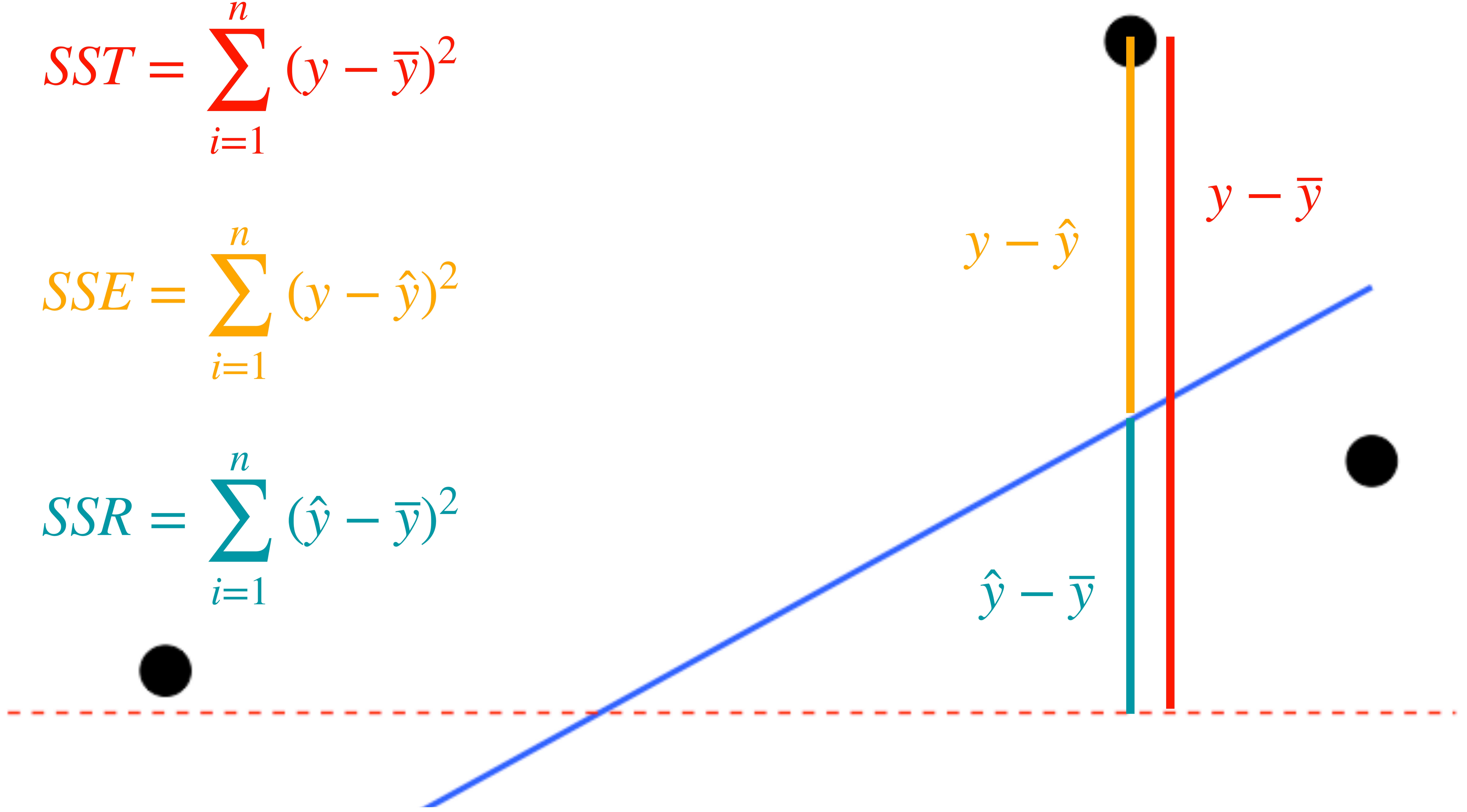
Model Validation:

What is r^2 **?**

$$SST = \sum_{i=1}^{n} (y - \bar{y})^2$$

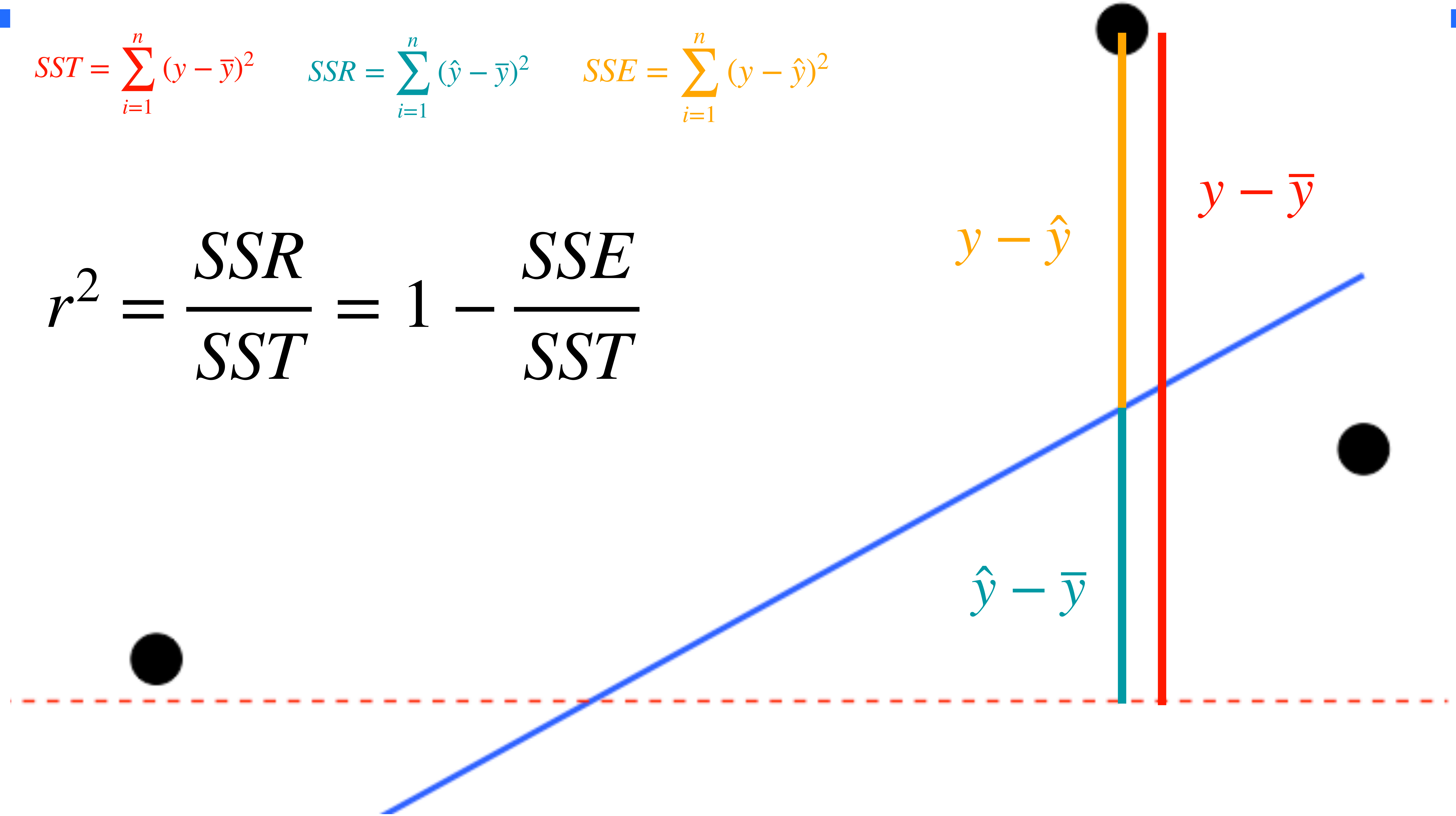
$$SSE = \sum_{i=1}^{n} (y - \hat{y})^2$$

$$SSR = \sum_{i=1}^{n} (\hat{y} - \bar{y})^2$$



$$SST = \sum_{i=1}^{n} (y - \bar{y})^2 \qquad SSR = \sum_{i=1}^{n} (\hat{y} - \bar{y})^2 \qquad SSE = \sum_{i=1}^{n} (y - \hat{y})^2$$

$$r^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST}$$

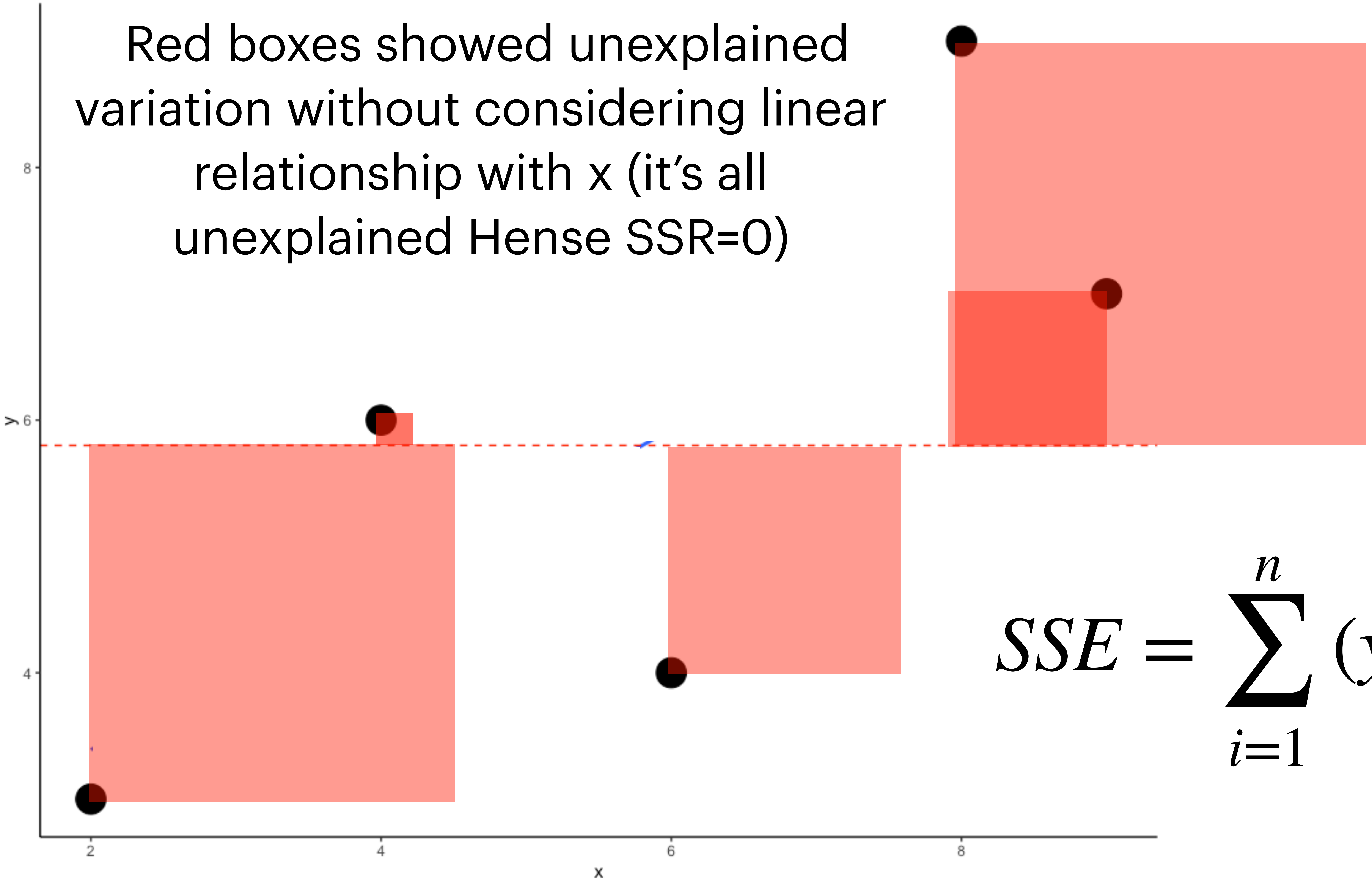


$$SST = \sum_{i=1}^{n} (y - \bar{y})^2 \qquad SSR = \sum_{i=1}^{n} (\hat{y} - \bar{y})^2 \qquad SSE = \sum_{i=1}^{n} (y - \hat{y})^2$$

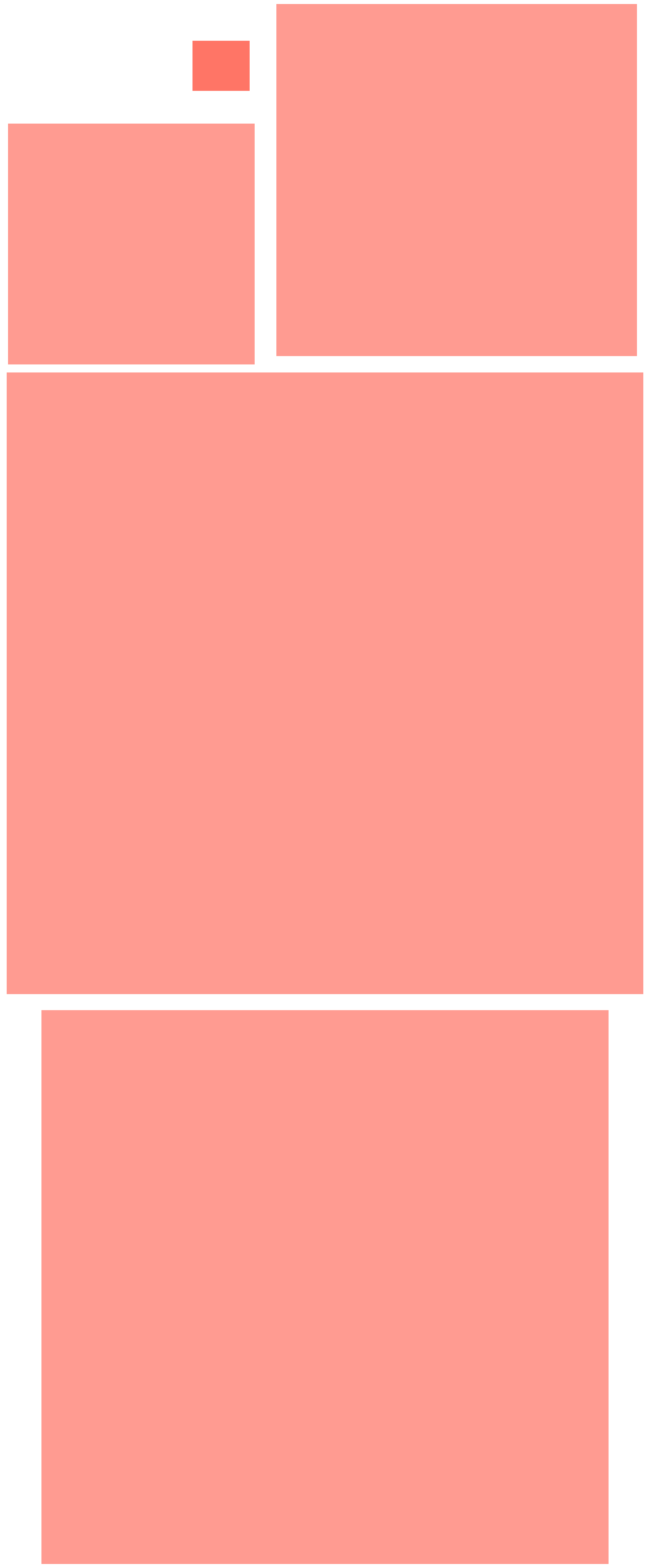
$$r^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST}$$

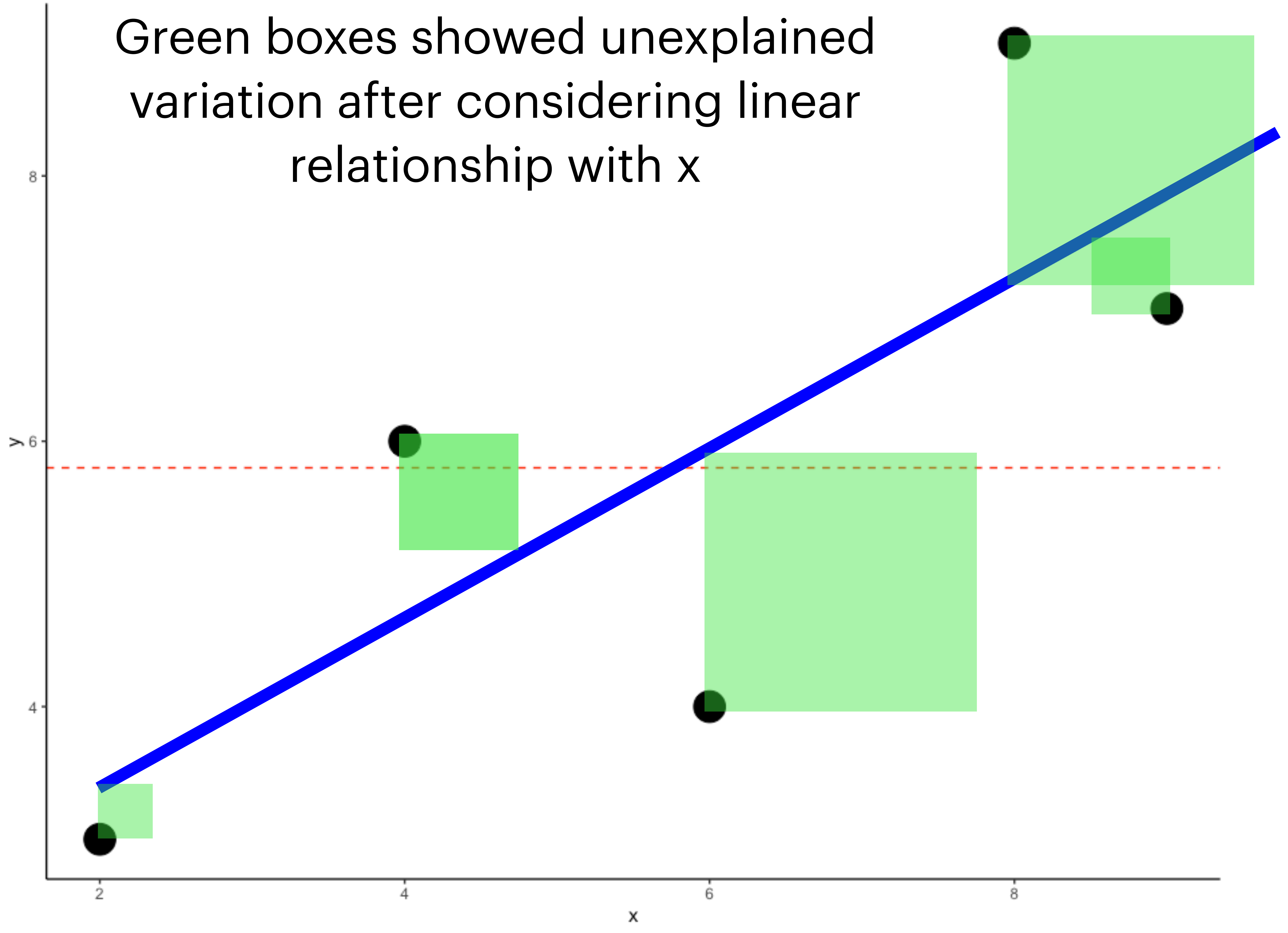
r^2 describes the percentage of variation in “y” that can be explained by “y’s” linear relationship with “x”

Red boxes showed unexplained variation without considering linear relationship with x (it's all unexplained Hense SSR=0)

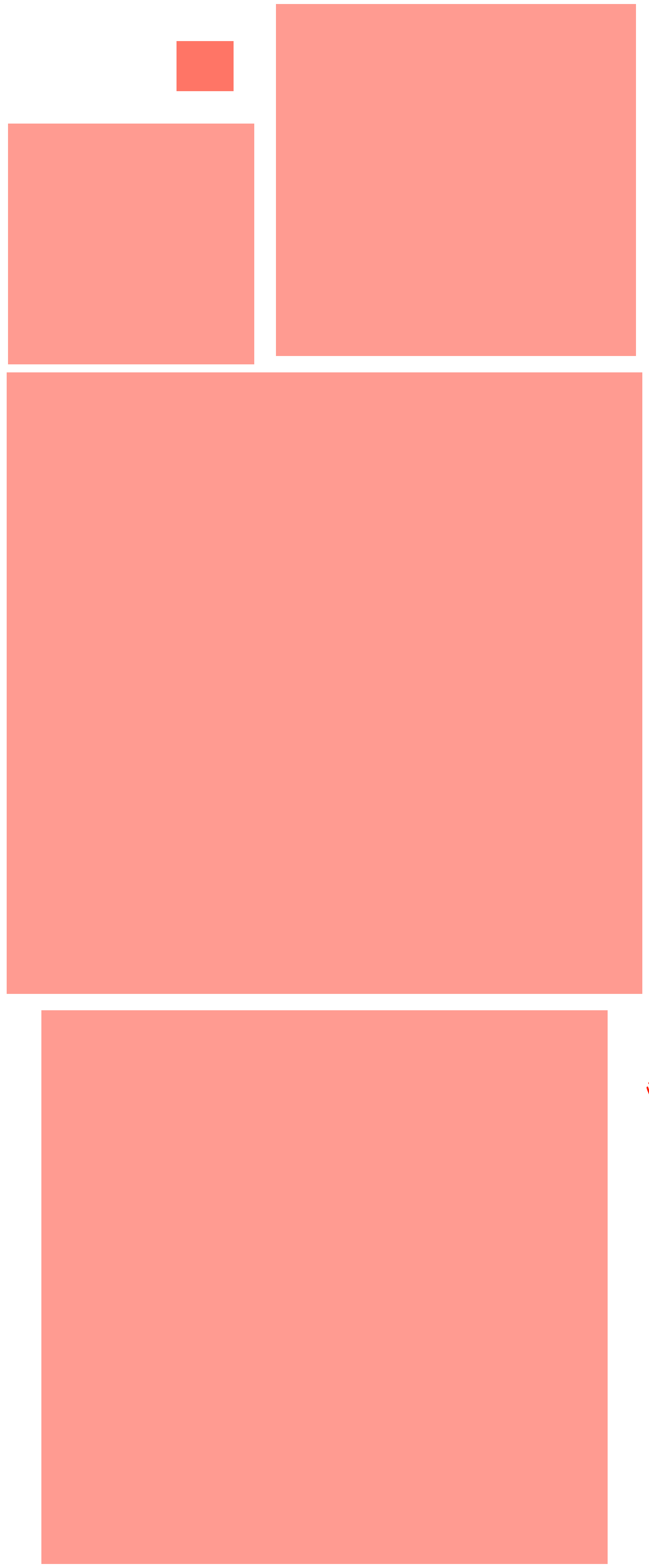


$$SSE = \sum_{i=1}^{n} (y - \hat{y})^2$$





$$SSE = \sum_{i=1}^{n} (y - \hat{y})^2$$



$$r^2 = \frac{SSR}{SST} = 1 - \frac{SSE}{SST}$$

When we consider the linear relationship the unexplained variance is reduced by $\frac{SSE}{SST}$ percent. The percentage “explained” by the model is $\frac{SSR}{SST}$

$$SSE = \sum_{i=1}^{n} (y - \hat{y})^2$$

Standard Error of the regression Line tells us the average residual length, in other words the average amount our model over/under predicts.

$$s = \sqrt{\frac{SSE}{n-2}}$$

Not expected to calculate by hand

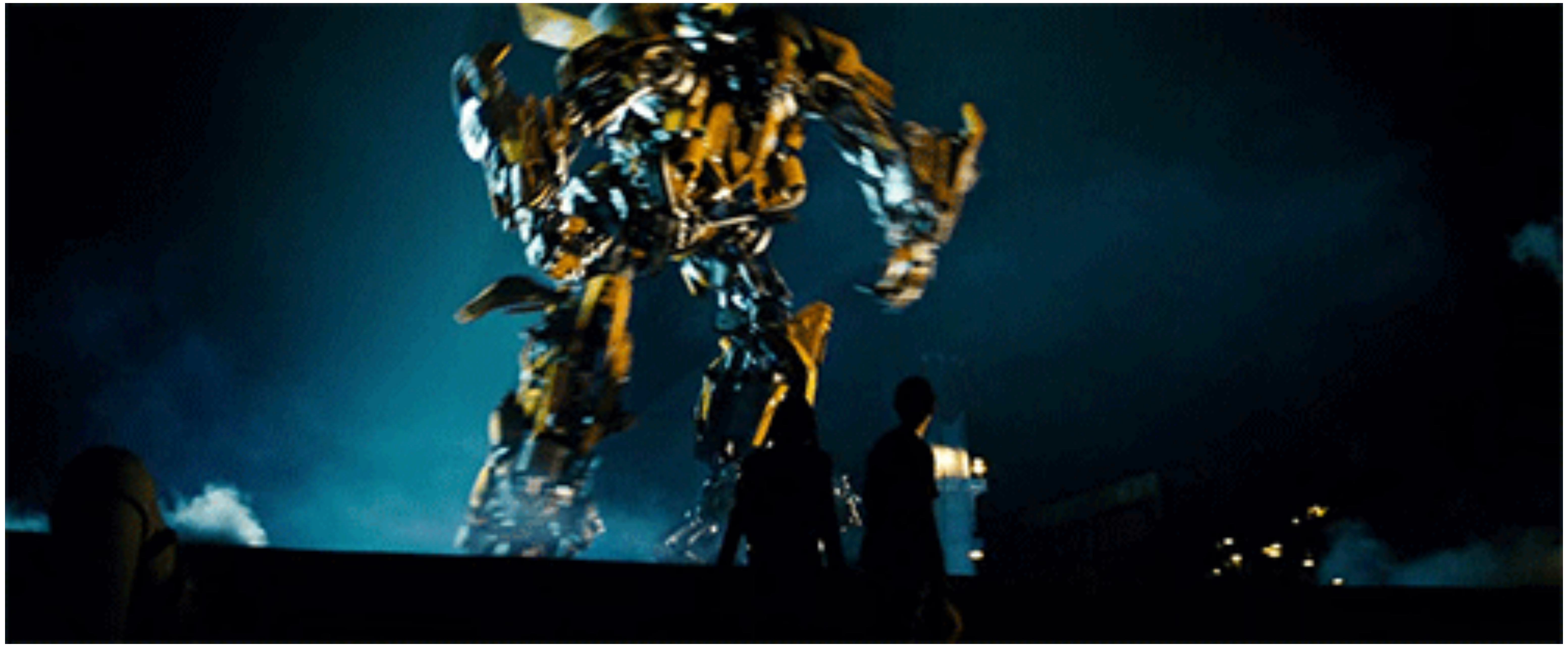
Examples:

- Deep Thoughts Unit 2 Q1-Q4
- Question 1 Page 143

Homework: Read Pages 113-130 Barron's, Quiz 8, Quiz 9

Transformations

Scatter Plot is Non-Linear



Linear Model Appropriate

There are many different transformations we might use

Method	Transform	Regression equation	Predicted value ($\hat{y}$)
Standard linear regression	None	$y = b_0 + b_1x$	$\hat{y} = b_0 + b_1x$
Exponential model	DV = log(y)	$\log(y) = b_0 + b_1x$	$\hat{y} = 10^{b_0 + b_1x}$
Quadratic model	DV = sqrt(y)	$\mathrm{sqrt}(y) = b_0 + b_1x$	$\hat{y} = (b_0 + b_1x)^2$
Reciprocal model	DV = 1/y	$1/y = b_0 + b_1x$	$\hat{y} = 1 / (b_0 + b_1x)$
Logarithmic model	IV = log(x)	$y= b_0 + b_1\log(x)$	$\hat{y} = b_0 + b_1\log(x)$
Power model	DV = log(y) IV = log(x)	$\log(y)=$ $b_0 + b_1\log(x)$	$\hat{y} = 10^{b_0 + b_1\log(x)}$

Transformations

Example: the length of a year for a planet, based on its distance from the sun. Here are the data:

Distance (millions of miles)	Year (# of Earth-years)
36	0.24
67	0.61
93	1
142	1.88
484	11.86
887	29.46
1784	84.07
2796	164.82
3666	247.68

Transformations

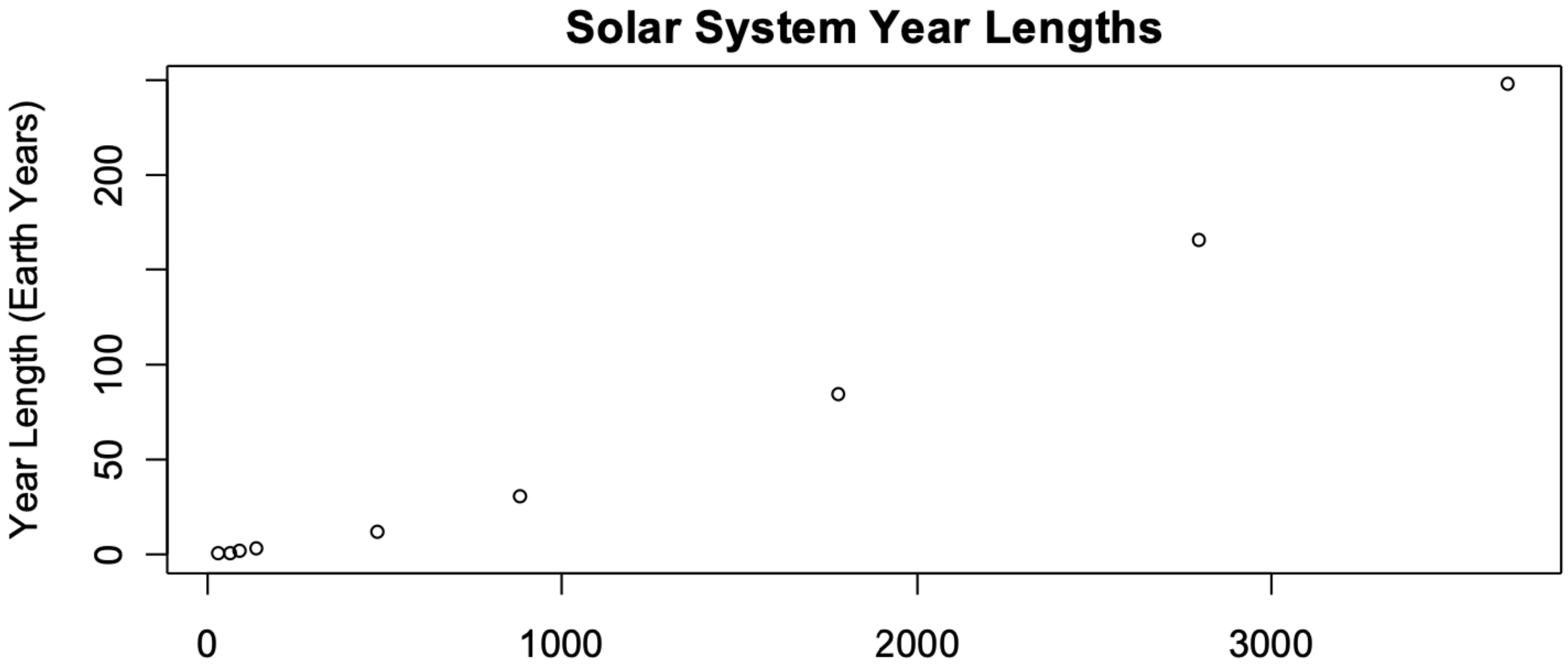
1. Let's run a simple linear regression.

What is r^2?

Is the Model Appropriate?

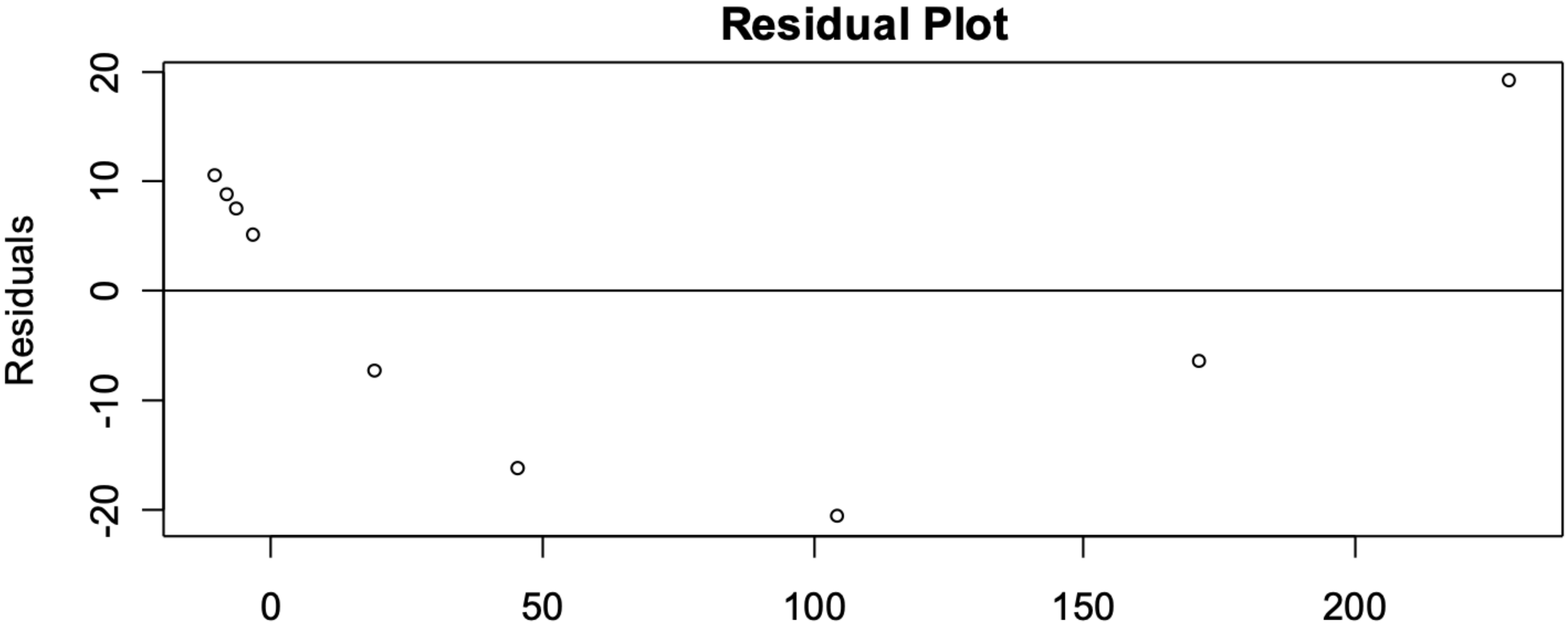
Transformations

Scatter Plot Looks non-linear



Transformations

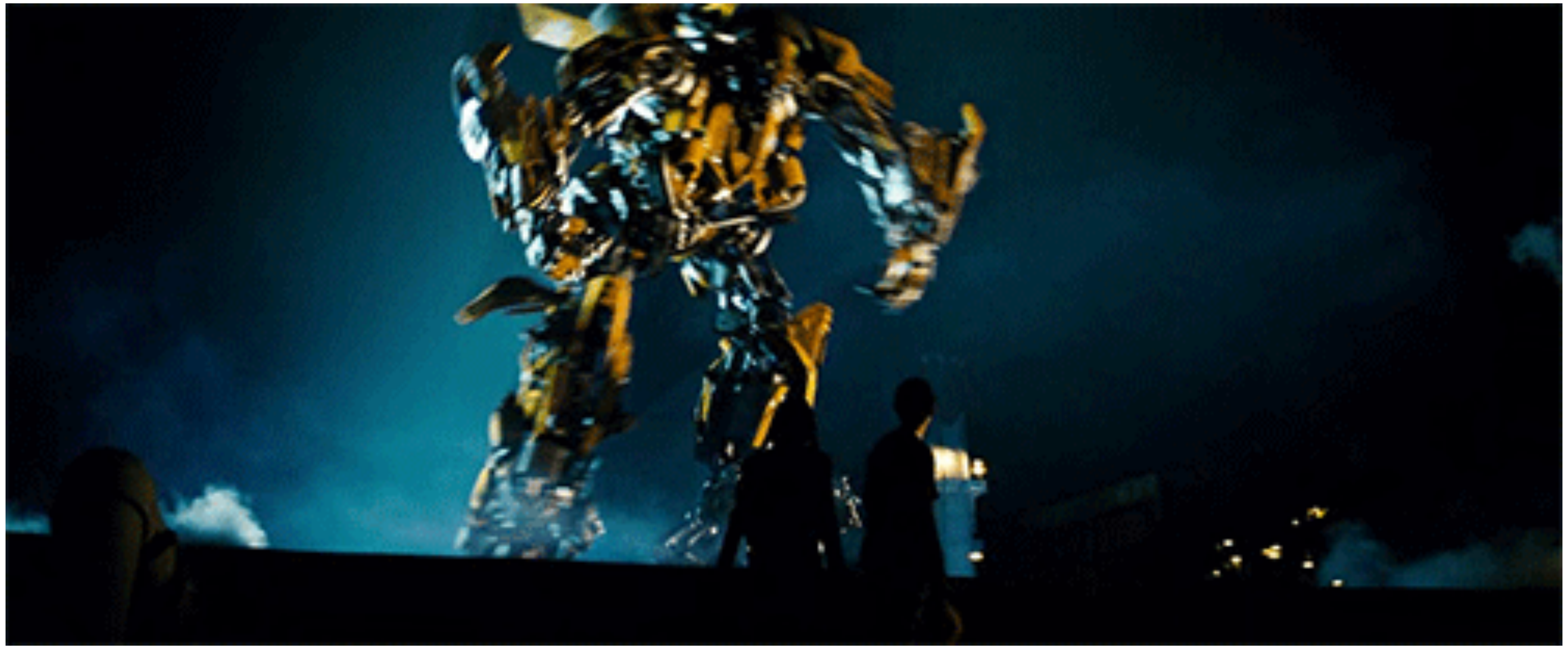
Residual plot makes non-linear pattern even more clear



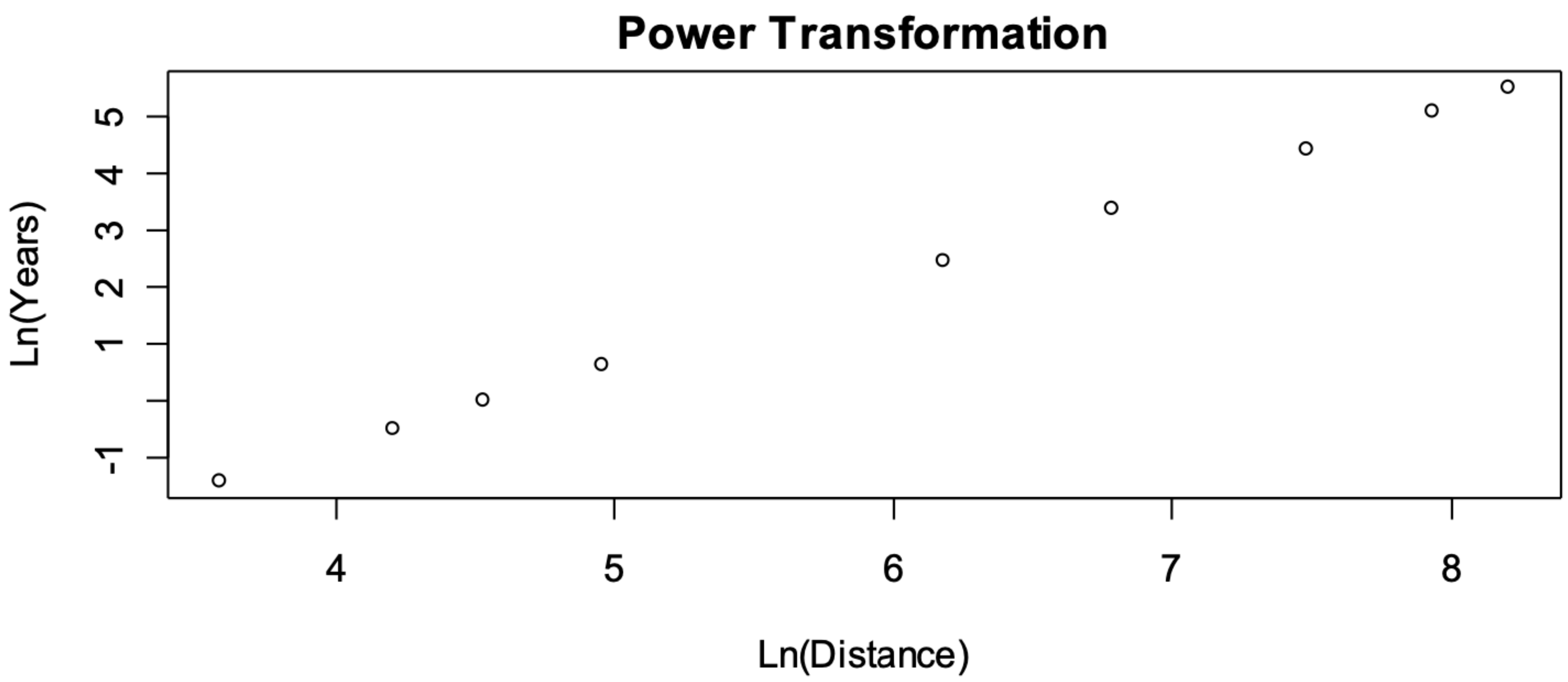
Transformations

1. Let's run a simple linear regression.

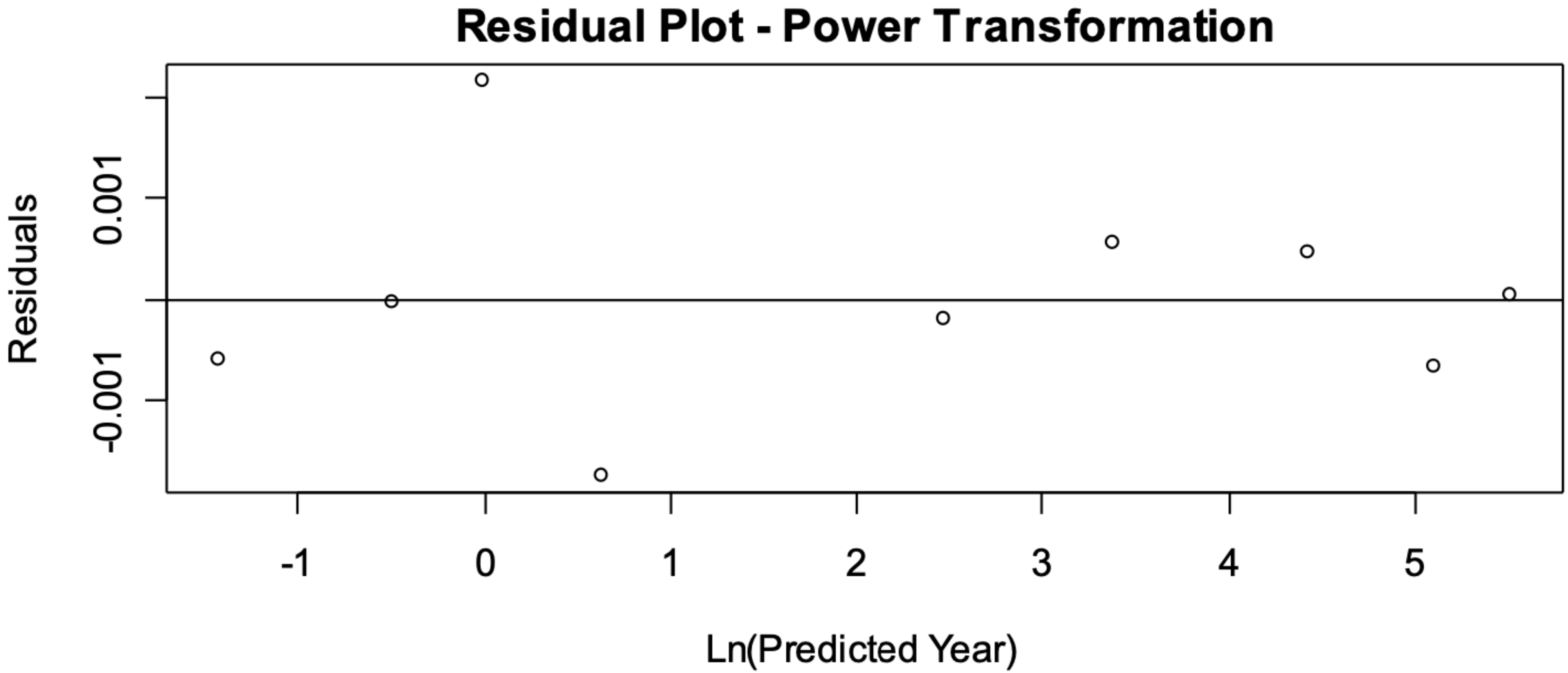
2. **Problem:** EW, that's not linear. Lets apply a power transformation



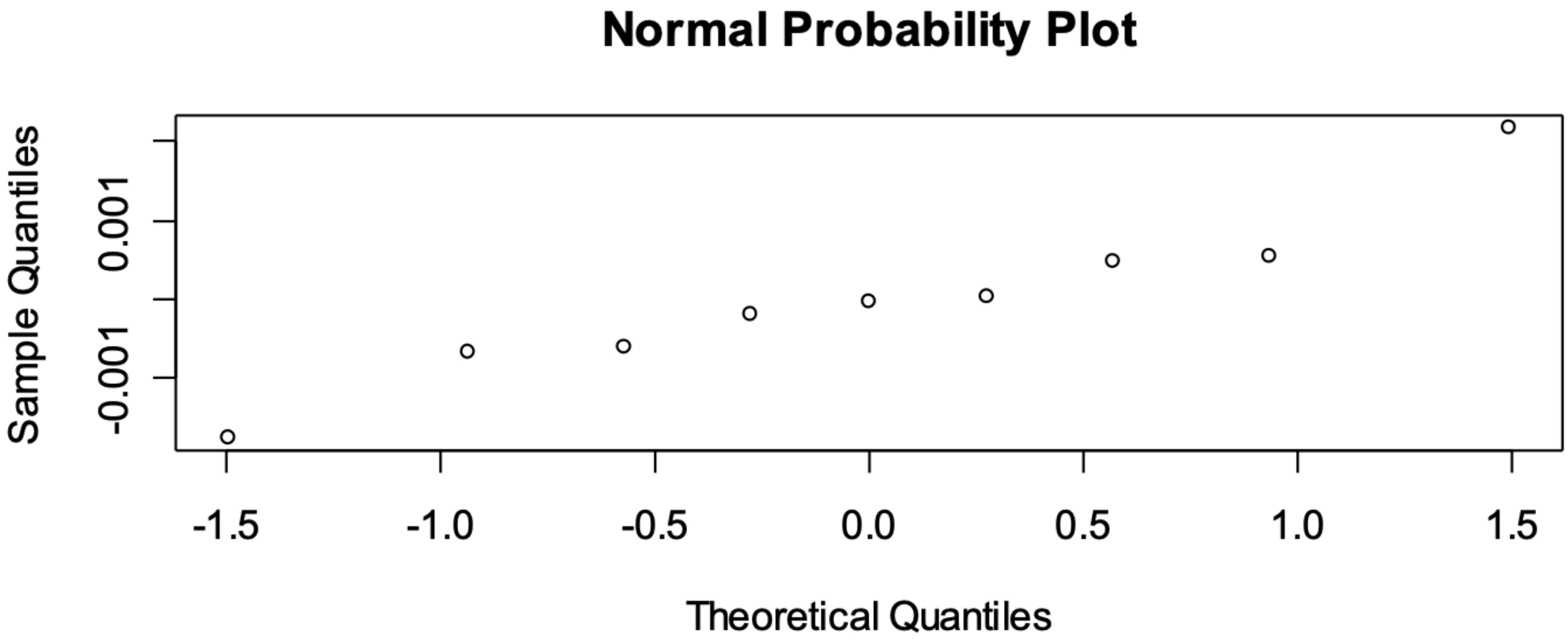
New Scatter Plot Looks much more linear



Residual plot improves significantly



Normality in residuals isn't bad either!



Transformations

1. Lets run a simple linear regression.

2. **Problem:** EW, that's not linear. Lets apply a power transformation

3. Run simple linear regression with transformed data.

4. Model: $ln(\hat{y}) = -6.8046 + 1.5008 \cdot ln(x)$

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Let's use this model to predict the year length of a planet that doesn't exist. The halfway point between Mars and Jupiter is around 313 million miles from Sol. What will this model predict for a year length if a planet occupied this position?

Model: $ln(\hat{y}) = -6.8046 + 1.5008 \cdot ln(x)$

$$ln(\hat{y}) = -6.8046 + 1.5008 \cdot ln(313)$$

$$ln(\hat{y}) = 1.8192$$

$$\hat{y} = e^{1.8192} = 6.167$$

What you need to know

- Recognize the need for a transformation
- Justify a transformations appropriateness

Examples:

- Barron’s pg. 130 Example 2.26
- Deep Thoughts Q5-Q6