Extra Practice

December 15, 2025

1. In a certain university 24% of the students are first year students. If a student is in their first year there is a 75% chance that they live on campus. If a student is not in their first year, there is only a 34% chance that they live on campus.

   (a) Draw a tree diagram to represent this problem.

   Solution: Let F = first-year and C = lives on campus. Use $P(F) = 0.24$, $P(\bar{F}) = 0.76$; $P(C \mid F) = 0.75$, $P(\bar{C} \mid F) = 0.25$; $P(C \mid \bar{F}) = 0.34$, $P(\bar{C} \mid \bar{F}) = 0.66$.

   (b) Use your tree diagram to construct a contingency table for this scenario.

   Solution: Joint probabilities:

$$P(F \cap C) = 0.24(0.75) = 0.18, \quad P(F \cap \bar{C}) = 0.24(0.25) = 0.06,$$

$$P(\bar{F} \cap C) = 0.76(0.34) = 0.2584, \quad P(\bar{F} \cap \bar{C}) = 0.76(0.66) = 0.5016.$$

	C	$\bar{C}$	Total
F	0.18	0.06	0.24
$\bar{F}$	0.2584	0.5016	0.76
Total	0.4384	0.5616	1

   (c) If a randomly selected student lives on campus, what is the probability that they are not in their first year?

   Solution:

$$P(\bar{F} \mid C) = \frac{P(\bar{F} \cap C)}{P(C)} = \frac{0.2584}{0.4384} \approx 0.5894.$$

   (d) Is being a first-year student independent of living on campus? Show why/why not.

   Solution: Not independent because $P(C \mid F) = 0.75 \neq P(C) = 0.4384$. (Equivalently, $P(F \cap C) = 0.18 \neq P(F)P(C) = 0.24(0.4384) = 0.105216$.)

2. Given that a person is "feeling lazy" there is a 15% chance that they work out. On days where the person isn't "feeling lazy", there is a 30% chance that they do not work out. There is a 10% chance that the person is "feeling lazy" on a given day.

   (a) What is the probability that the person is not feeling lazy, given that they do not work out?

   Solution: Let L = lazy and W = works out.

$$P(L) = 0.10,\ P(\bar{L}) = 0.90,\ P(W \mid L) = 0.15 \Rightarrow P(\bar{W} \mid L) = 0.85,\ P(\bar{W} \mid \bar{L}) = 0.30.$$

$$P(\bar{W}) = 0.10(0.85) + 0.90(0.30) = 0.355.$$

$$P(\bar{L} \mid \bar{W}) = \frac{P(\bar{L} \cap \bar{W})}{P(\bar{W})} = \frac{0.90(0.30)}{0.355} = \frac{0.27}{0.355} \approx 0.7606.$$

   (b) Assuming that days are independent, how many days is the person expected to work out over 10 days?

   Solution: First, $P(W \mid \bar{L}) = 1 - 0.30 = 0.70.$

$$P(W) = 0.15(0.10) + 0.70(0.90) = 0.645.$$

   Expected workouts in 10 days: $10(0.645) = 6.45$.

   (c) Let X be the number of days that the person works out over a 4 day period. Find each of the following:

      (a) $P(X \geq 1)$

      Solution: With independence, $X \sim \text{Bin}(4, 0.645)$.

$$P(X \geq 1) = 1 - P(X = 0) = 1 - (0.355)^4.$$

      (b) $P(X = 3)$

      Solution:

$$P(X = 3) = \binom{4}{3}(0.645)^3(0.355).$$

      (c) $Var(X)$

      Solution:

$$Var(X) = np(1-p) = 4(0.645)(0.355) = 0.9159.$$

Suppose that the number of pieces of Wow Chicken Brenden eats, C, on a given visit has the following probability distribution.

c	0	1	2	3	4	5	6	7	8
$P(C = c)$	0.01	0.1	0.2	0.3	0.3	0.01	0.01	0.01	0.06

You may assume that Brenden's visits to Wow Chicken are independent.

3. What is $E[C]$?

Solution:

$$E[C] = 3.26$$

4. What is $Var[C]$?

Solution:

$$Var[C] = 2.7124$$

Now suppose that each piece of chicken costs \$1.50, and Brenden always gives a \$4.50 tip. Let M be the amount of money Brenden spends.

5. What is $E[M]$?

Solution:

$$E[M] = 1.50E[C] + 4.50 = 9.39$$

6. What is $Var[M]$?

Solution:

$$Var[M] = (1.5)^2 Var[C] = 6.1029$$

7. Suppose Brenden visits Wow Chicken 4 successive days in a row. Assume days are independent. What is the probability he eats exactly one piece of chicken on one day and two or more pieces of chicken on all other days?

Solution:

$$P(C = 1) = 0.10, \qquad P(C \geq 2) = 1 - (0.01 + 0.10) = 0.89.$$

Choose which day is the "1-piece" day:

$$\binom{4}{1}(0.10)(0.89)^3.$$

Suppose that historically Izaiah wins $\frac{4}{9}$ of the Siege matches he plays.

8. Let X be the number of matches it takes for Izaiah to win, if he plays matches until he wins.

   (a) $P(X > 2)$

   Solution: Geometric with $p = \frac{4}{9}$, $q = \frac{5}{9}$:

   $$P(X > 2) = q^2 = \left(\frac{5}{9}\right)^2 = \frac{25}{81}.$$

   (b) $P(X = 2 \cup X = 3)$

   Solution:

   $$P(X = 2) = qp = \frac{5}{9} \cdot \frac{4}{9} = \frac{20}{81}, \qquad P(X = 3) = q^2 p = \left(\frac{5}{9}\right)^2 \frac{4}{9} = \frac{100}{729}.$$

   $$P(X = 2 \cup X = 3) = \frac{20}{81} + \frac{100}{729} = \frac{280}{729}.$$

   (c) $E(X)$

   Solution:

   $$E(X) = \frac{1}{p} = \frac{9}{4}.$$

   (d) $Var(X)$

   Solution:

   $$Var(X) = \frac{1-p}{p^2} = \frac{q}{p^2} = \frac{5/9}{(4/9)^2} = \frac{45}{16}.$$

   (e) What assumptions are we making in order to use the model in parts a–d?

   Solution: Matches are independent and the win probability stays constant at $p = \frac{4}{9}$ from match to match.

9. Suppose Izaiah plays n games.

(a) How many of the n games is Izaiah expected to win?

Solution: If $W \sim \text{Bin}(n, \frac{4}{9})$, then $E[W] = n \cdot \frac{4}{9} = \frac{4n}{9}$.

(b) Izaiah starts by eating 4 Oreo cookies and rewards himself with two Oreo cookies for every game he wins. How many Oreo cookies should he expect to eat if he plays 50 games?

Solution: Let $W \sim \text{Bin}(50, \frac{4}{9})$. Cookies $= 4 + 2W$.

$$E[4+2W] = 4 + 2E[W] = 4 + 2\left(50 \cdot \frac{4}{9}\right) = 4 + \frac{400}{9} = \frac{436}{9}.$$

(c) What is the probability Izaiah eats less than 7 Oreo cookies?

Solution: Cookies $= 4 + 2W < 7 \iff W \leq 1$.

$$P(W \leq 1) = \left(\frac{5}{9}\right)^{50} + 50\left(\frac{4}{9}\right)\left(\frac{5}{9}\right)^{49}.$$

10. Dr. Vince is training for the math olympiad. His historic average is 99 questions correct out of every 100. For this problem assume questions are independent.

   (a) When practicing, what is the probability that he doesn't get a question correct until his 5th attempted problem?

   Solution: With $p = 0.99$ and $q = 0.01$,

   $$P(\text{first correct on 5th}) = q^4 p = (0.01)^4(0.99) = 9.9 \times 10^{-9}.$$

   (b) When completing a 20 question competition, what is the probability Dr. Vince gets more than 4 questions correct?

   Solution: If $X \sim \text{Bin}(20, 0.99)$, then

   $$P(X > 4) = 1 - \sum_{k=0}^{4} \binom{20}{k} (0.99)^k (0.01)^{20-k}.$$

   (c) Suppose Dr. Vince eats 2 chocolate muffins for every problem he gets right and does 50 pushups for every question he gets wrong. Every muffin contains 53 calories and pushups burn 7 calories per 10 pushups. If Dr. Vince completes 100 problems, what is his expected net calorie change?

   Solution: Let $X \sim \text{Bin}(100, 0.99)$ be the number correct, so $E[X] = 99$. Muffins: $2X$ muffins $\Rightarrow 106X$ calories. Wrong: $100 - X$ wrong $\Rightarrow 50(100 - X)$ pushups. Burn rate: 7 calories per 10 pushups $\Rightarrow 0.7$ cal/pushup, so burned $= 0.7 \cdot 50(100 - X) = 35(100 - X)$. Net:

   $$C = 106X - 35(100 - X) = 141X - 3500.$$

   $$E[C] = 141E[X] - 3500 = 141(99) - 3500 = 10459.$$

   (d) Let C be the net calorie change in part (c). Determine $Var(C)$.

   Solution: Since $C = 141X - 3500$,

   $$Var(C) = 141^2 Var(X).$$

   For $X \sim \text{Bin}(100, 0.99)$,

   $$Var(X) = npq = 100(0.99)(0.01) = 0.99.$$

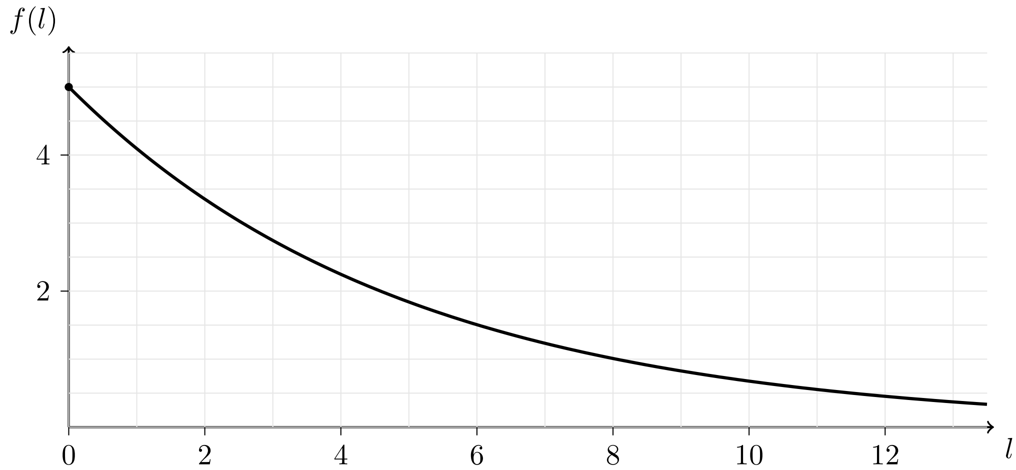
   Thus

   $$Var(C) = 141^2(0.99) = 19881(0.99) = 19682.19.$$

   (e) Are the assumptions made in this problem realistic? Why or why not?

   Solution: Not fully: the model assumes independence and constant success probability $p = 0.99$ across many questions; fatigue, changing difficulty, and learning can violate these.

11. Brenden's late time on Friday morning's class follows an exponential distribution. Let L be Brenden's late time. The probability density function $f(l)$ is shown below.



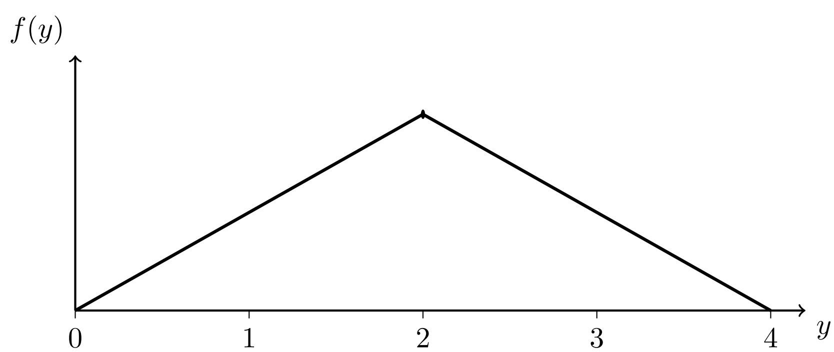
(a) Shade the region that represents $P(L \geq 12)$

Solution: Shade the area under $f(l)$ from $l = 12$ to the right.

(b) Shade the region that represents $P(2 \leq L \leq 6)$

Solution: Shade the area under $f(l)$ from $l = 2$ to $l = 6$.

12. The graph below represents the pdf $f(y)$ for a random variable Y.



(a) Fill in piecewise function for the pdf below:

$$f(y) = \begin{cases} & 0 \le y \le 2 \\ & 2 \le y \le 4 \\ 0 & \text{otherwise} \end{cases}$$

Solution: Total area is 1. With base 4, peak height h satisfies $\frac{1}{2}(4)(h) = 1 \Rightarrow h = \frac{1}{2}$. Thus

$$f(y) = \begin{cases} \dfrac{y}{4} & 0 \le y \le 2 \\ \dfrac{4-y}{4} & 2 \le y \le 4 \\ 0 & \text{otherwise.} \end{cases}$$

(b) Determine $P(Y < 1)$

Solution:

$$P(Y < 1) = \int_0^1 \frac{y}{4}\, dy = \left.\frac{y^2}{8}\right|_0^1 = \frac{1}{8}.$$

(c) Determine $P(1 < Y < 3)$

Solution:

$$P(1 < Y < 3) = \int_1^2 \frac{y}{4}\, dy + \int_2^3 \frac{4-y}{4}\, dy = \left(\frac{4-1}{8}\right) + \left(\frac{4-1}{8}\right) = \frac{3}{8} + \frac{3}{8} = \frac{3}{4}.$$

13. (1 point) The diastolic pressure among 20 to 30 year olds is roughly normal. If 10 percent have levels above 86 mmHg, and 20 percent have levels below 69 mmHg, what is the mean of this distribution?

    A. 74.5 mmHg

    B. 75.7 mmHg

    C. 77.5 mmHg

    D. 79.3 mmHg

    E. The mean cannot be calculated from the given information

    Solution: Let $X \sim N(\mu, \sigma)$. Given $P(X > 86) = 0.10$, so $86 = \mu + z_{0.90}\sigma$, and $P(X < 69) = 0.20$, so $69 = \mu + z_{0.20}\sigma$.

    Using $z_{0.90} \approx 1.28$ and $z_{0.20} \approx -0.84$:

$$86 - 69 = (1.28 - (-0.84))\sigma \Rightarrow \sigma \approx 8.02,$$

$$\mu = 86 - 1.28(8.02) \approx 75.7.$$

14. (1 point) Among 125 teachers at a small college, 75 are registered Democrats, 35 are registered Republicans, and the rest are independents. If a 10 person committee is randomly picked, what is the probability that at least two independents are chosen?

    A. $\binom{10}{2}\left(\frac{15}{125}\right)^2\left(\frac{110}{125}\right)^8$

    B. $1 - \left[\left(\frac{15}{125}\right)^2 + \left(\frac{15}{125}\right)^1 + \left(\frac{15}{125}\right)^0\right]$

    C. $1 - \left[\frac{\binom{110}{10}}{\binom{125}{10}} + \frac{\binom{15}{1}\binom{110}{9}}{\binom{125}{10}}\right]$

    D. $\frac{\binom{15}{2}\binom{110}{8}}{\binom{125}{10}}$

    E. None of these is correct

    Solution: There are 15 independents and 110 non-independents.

    "At least two independents" $= 1 -$ (0 or 1) independent:

$$1 - \left[\frac{\binom{110}{10}}{\binom{125}{10}} + \frac{\binom{15}{1}\binom{110}{9}}{\binom{125}{10}}\right].$$

15. (1 point) In a set of eight boxes, three boxes each contain two red and two green marbles, while the remaining boxes each contain three red and two green marbles. A player randomly picks a box and then randomly picks a marble from that box. She wins if she ends up with a red marble. If she plays four times, what is the probability that she wins exactly twice?

    A. 0.0606

    B. 0.3164

    C. 0.3221

D. 0.3634

E. 0.5625

Solution: Probability of red from a randomly chosen box:

$$P(R) = \frac{3}{8}\left(\frac{2}{4}\right) + \frac{5}{8}\left(\frac{3}{5}\right) = 0.525.$$

With four independent plays, $X \sim \mathrm{Bin}(4, 0.525)$.

$$P(X = 2) = \binom{4}{2}(0.525)^2(0.475)^2 \approx 0.3634.$$

16. (1 point) Three fair coins are tossed. If all lands “heads,” the player wins \$10, and if exactly two land heads, the player wins \$5. If it costs \$4 to play, what is the players expected outcome after four games?

A. Loss of \$0.875

B. Loss of \$1.00

C. Loss of \$3.50

D. Loss of \$2.25

E. Win of \$9.00

Solution: Single game expected value:

$$E = 10\left(\frac{1}{8}\right) + 5\left(\frac{3}{8}\right) - 4 = 1.25 + 1.875 - 4 = -0.875.$$

Over four games:

$$4(-0.875) = -3.50.$$