1. Suppose the weights of salmon are normally distributed with a mean of 19.2 lbs and a standard deviation of 2.2 lbs. On a fishing trip you catch 10 salmon. (You may assume each salmon is caught randomly and is independent of each other).

   (a) What is the probability that the total weight of the salmon exceeds 200 lbs?

   – let $X_1, X_2, \dots, X_{10}$ be the weights of salmon.

   – $X_i \sim$ normal $(\mu = 19.2, \sigma = 2.2)$

   – $X_1 + X_2 + \dots + X_{10} \sim$ normal $(\mu = 192, \sigma = \sqrt{10}\cdot 2.2)$

   $P(T > 200) =$ normalcdf $(200, \infty, 192, \sqrt{10}\cdot 2.2) = \boxed{0.125}$

   $E(X_1 + X_2 + \dots + X_{10})$
   $= E(X_1) + E(X_2) + \dots + E(X_{10})$

   $Var(X_1 + X_2 + \dots + X_{10})$
   $= \underbrace{Var(X_1)}_{2.2^2} + \underbrace{Var(X_2)}_{2.2^2} + \dots + \underbrace{Var(X_{10})}_{2.2^2}$
   $= 10 \cdot 2.2^2$

   $SD(X_1 + X_2 + \dots + X_{10}) = \sqrt{10}\cdot 2.2$

   (b) What is the variance for the total weight of the salmon?

   $10 \cdot 2.2^2$ lb^2

   (c) Suppose you only keep salmon that weigh more than ~~30~~ $\boxed{23}$ lbs. What is the probability that the third salmon you keep is the 10^{th} that you have caught?

   $P(\text{Keep}) =$ normalcdf $(23, \infty, 19.2, 2.2) = p = 0.042$

   $P(\text{Keep}^c) = (1-p)$

   $= \binom{9}{2} p^3 (1-p)^7 = \boxed{0.0019827}$

   $\overbrace{\underline{\times}\ \underline{\times}\ \underline{\checkmark}\ \underline{\times}\ \underline{\times}\ \underline{\checkmark}\ \underline{\times}\ \underline{\times}\ \underline{\times}}\ \underset{p}{\underline{\checkmark}}$

2. A fish tank at the pet store is filled with 30 guppies. 20 of them are red and 10 of them are blue. You scoop out 3 guppies at random.

   (a) What is the probability that exactly two of the three guppies are red?

   $X \sim$ hypergeometric

   $P(X=2) = \dfrac{\binom{20}{2}\binom{10}{1}}{\binom{30}{3}} = \boxed{0.4679803}$

   (b) The pet store charges \$2.50 for red guppies and \$1.50 for blue guppies. Find the expected price for the three guppies.

   $P(X=0) = \dfrac{\binom{10}{3}}{\binom{30}{3}}$

   $P(X=2) = \dfrac{\binom{20}{2}\binom{10}{1}}{\binom{30}{3}} = 0.4679803$

   $P(X=1) = \dfrac{\binom{20}{1}\binom{10}{2}}{\binom{30}{3}}$

   $= 0.2216749$

   $P(X=3) = \dfrac{\binom{20}{3}}{\binom{30}{3}} = 0.2807882$

   $E(X) = P(1) + 2P(2) + 3P(3)$

   $= 3\left(\frac{2}{3}\right) = 2$

   <u>let P be price you pay</u>

   $P = 2.5X + 1.5(3 - X)$

   $= 2.5X + \frac{9}{2} - 1.5X$

   $= X + \frac{9}{2}$

   $E(P) = E(X) + (9/2)$

   $= 2 + \frac{9}{2}$

   $\boxed{= \$6.5}$

 Note the order of questions was edited.

1. Megan and Tal played numerous games of chess and it was determined that the probability that Megan would win is 0.40, the probability that Tal would win is 0.35, and the probability that the game would end in a draw is 0.25. Suppose Tal and Megan play 6 games of chess.

   (a) What is the probability that Megan does not win until after her 4^{th} game?

   let X be the # of games Megan plays before she wins

   $X \sim \text{geometric}(p=0.4)$

   $P(X=5)+P(X=6)+\cdots = 1-(P(X=1)+P(X=2)+P(X=3)+P(X=4)) = 1-(0.4+0.4(0.6)+0.4(0.6)^2+0.4(0.6)^3)$

   $\boxed{=0.1296}$

   (b) Given that there are 2 ties, what is the probability that Tal wins more than 2 games?

   $P(\text{Tal wins} > 2 \text{ games} \mid 2 \text{ ties}) = \dfrac{P(\text{Tal wins} > 2 \text{ games} \cap 2 \text{ ties})}{P(2 \text{ ties})}$

   $= \dfrac{\underline{\text{tie}}\ \underline{\text{tie}}\ \underline{\text{win}}\ \underline{\text{win}}\ \underline{\text{win}}\ \underline{\text{lose}} + \underline{\text{tie}}\ \underline{\text{tie}}\ \underline{\text{win}}\ \underline{\text{win}}\ \underline{\text{win}}\ \underline{\text{win}}}{\underline{\text{tie}} \cdot \underline{\text{tie}}\ \underline{\text{tie}^c}\ \underline{\text{tie}^c}\ \underline{\text{tie}^c}\ \underline{\text{tie}^c}} = \dfrac{\left(\frac{6!}{2!\,3!}\right)(0.25)^2(0.35)^3(0.4) + \left(\frac{6!}{2!\,4!}\right)(0.25)^2(0.35)^4}{\binom{6}{2}(0.25)^2(0.75)^4}$

   $\boxed{=0.264237}$

   (c) What is the probability that in the 6 games Megan wins 2, Tal wins 2, and there are 2 draws?

   $\left(\frac{6!}{2!\,2!\,2!}\right)\underline{0.4}\ \underline{0.4}\ \underline{0.35}\ \underline{0.35}\ \underline{0.25}\ \underline{0.25} = \boxed{0.11025}$

   (d) What is the probability that there are exactly 3 ties?

   $\underline{\text{tie}}\ \underline{\text{tie}}\ \underline{\text{tie}}\ \underline{\text{tie}^c}\ \underline{\text{tie}^c}\ \underline{\text{tie}^c}$

   let Y be # of ties.

   $Y \sim \text{binomial}(n=6, p=0.25)$

   $P(Y=3) = \binom{6}{3}(0.25)^3(0.75)^3 = \boxed{0.1318}$

(e) What is the expected number of ties.

$$E(Y) = 6(0.25) = \boxed{1.5 \text{ ties}}$$

(f) Tal does 10 pushups everytime he loses, 2 pushups everytime he ties, and no pushups when he wins. Over the 6 games how many pushups is he expected to do?

Method #1

p	0	2	10
$P(p)$	0.35	0.25	0.40

Tal expects to do $2(0.25)+10(0.4)$
$= 0.5 + 4$
$= 4.5$ pushups per game.

So he is expected to do $\boxed{6(4.5) = 27}$ pushups over 6 independent games.

Method #2.

- Let X_1 be the number of Tal's Loses $X_1 \sim \text{binomial}(6, 0.40)$
- Let X_2 be the number of Tal's ties. $X_2 \sim \text{binomial}(6, 0.25)$
- X_1 and X_2 are NOT independent.
- Let T be the number of pushups Tal does.

$T = 10X_1 + 2X_2$

$E(T) = 10E(X_1) + 2E(X_2)$

$= 10 \cdot 6 \cdot (0.4) + 2 \cdot 6 \cdot 0.25$

$= 24 + \frac{12}{4} = \boxed{27 \text{ pushups.}}$

(g) Tal is determined to keep playing until he wins 5 games. How many games is he expected to play?

Reword to match 6 games

Let N be the number of games played before Tal wins one

$N \sim \text{geometric}(0.35)$

$$E(N) = \frac{1}{0.35} = \frac{100}{35} = \frac{20}{7} = \boxed{2\tfrac{6}{7} \text{ games}}$$

This means Tal is expected to play before winning 5 $\quad 5\left(\frac{20}{7}\right) = \frac{100}{7} = \boxed{14\tfrac{2}{7} \text{ games}}$

(h) What is the probability that Megan wins her third game on the 6^{th} that they play?

$$\underbrace{0.4}_{\text{win}} \;\; \underbrace{0.4}_{\text{win}} \;\; \underbrace{0.6 \;\; 0.6 \;\; 0.6}_{\text{no wins}} \;\; \Big| \;\; \underbrace{0.4}_{\text{win}}$$

fixed

$$\binom{5}{3}(0.4)^3(0.6)^3 = \boxed{0.13824}$$

Do Multinomial probabilities sum to binomial?

$$\binom{6}{2} p_t^2 (1-p_t)^4 \quad \text{- result.}$$

$$\frac{6!}{l!\,2!\,w!}(p_t)^2 (p_l)^l (p_w)^w \quad \text{for all} \quad l+w=4$$

l	w
0	4
1	3
2	2
3	1
4	0

$$\left[\left(\frac{6!}{2!\,4!}\right) p_t^2 p_w^4 + \left(\frac{6!}{2!\,3!}\right) p_t^2 p_w^3 p_l^1 + \left(\frac{6!}{2!\,2!\,2!}\right) p_t^2 p_w^2 p_l^2 + \left(\frac{6!}{2!\,3!}\right) p_t^2 p_w^1 p_l^3 + \left(\frac{6!}{2!\,4!}\right) p_t^2 p_l^4\right]$$

$$\binom{6}{2} p_t^2 \left[p_w^4 + 4 p_w^3 p_l^1 + 6 p_w^2 p_l^2 + 4 p_w^1 p_l^3 + p_l^4\right]$$

$$\binom{6}{2} p_t^2 \underbrace{(p_w + p_l)}_{p_t^c}{}^4$$

$$\frac{6!}{l!\,2!\,w!}(p_t)^2 (p_l)^l (p_w)^w = \binom{6}{2} p_t^2 \left(\frac{4!}{l!\,w!}\right) p_l^l p_w^w$$

we have $l+w=4$

$$= \binom{6}{2} p_t^2 \binom{4}{l} p_l^l p_w^{4-l}$$

Want to sum of all values of l.

$$\binom{6}{2} p_t^2 \sum_{l=0}^{4} \binom{4}{l} (p_l)^l (p_w)^{4-l} = \binom{6}{2} p_t^2 (p_w + p_l)^4$$

$$\boxed{= \binom{6}{2} p_t^2 (1-p_t)^4}$$