Probability and Sets Questions

1. A survey of a groups viewing habits over the last year revealed the following information:

   - 28% watched gymnastics
   - 29% watched baseball
   - 19% watched soccer
   - 14% watched gymnastics and baseball
   - 12% watched baseball and soccer
   - 10% watched gymnastics and soccer
   - 8% watched all three sports

   Calculate the percentage of the group that watched none of the three sports during the last year.

$P(A \cup B \cup C)^c = P(A^c \cap B^c \cap C^c)$

$P(A \cup B \cup C) =$

$=$

$P(G) = 0.28$

$P(B) = 0.29$

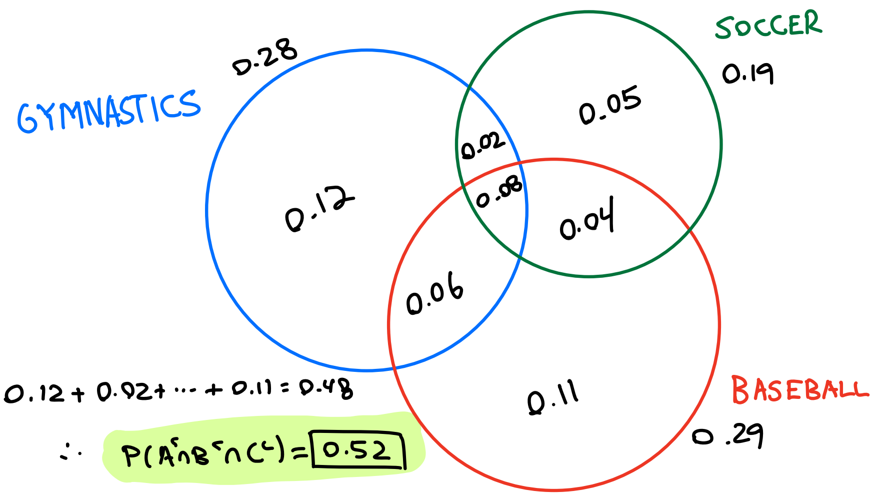
$P(S) = 0.19$

$P(G \cap B) = 0.14$

$P(B \cap S) = 0.12$

$P(G \cap S) = 0.10$

$P(G \cap S \cap B) = 0.08$



$0.12 + 0.02 + \cdots + 0.11 = 0.48$

$\therefore \; P(A^c \cap B^c \cap C^c) = \boxed{0.52}$

OR! $P(A \cup B \cup C) = P(A) + P(C) + P(B) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

2. The probability that a visit to a primary care physician's (PCP) office results in neither lab work nor referral to a specialist is 35%. Of those coming to a PCP's office, 30% are referred to specialists and 40% require lab work. Calculate the probability that a visit to a PCP's office results in both lab work and referral to a specialist.

$P(L^c \cap R^c) = 0.35 = P((L \cup R)^c)$ so $P(L \cup R) = 0.65$

$P(L) = 0.4$

$P(R) = 0.3$

$P(L \cup R) = P(L) + P(R) - P(L \cap R)$

$\boxed{P(L \cap R) = 0.4 + 0.3 - 0.65 = 0.05}$

3. $P(A \cup B) = 0.7$ and $P(A \cup B^c) = 0.9$. Calculate $P(A)$

$P(A \cup B) = 0.7$ $\quad$ $P(A^c \cap B) = 0.1$

$P(A^c \cap B^c) = 0.3$ $\quad$ $P(A^c \cap B^c) + P(A^c \cap B) = P(A^c)$

$= 0.4$

$\boxed{P(A) = 0.6}$

4. An auto insurance company has 10,000 policyholders. Each policyholder is classified as:

   (a) young or old;
   (b) male or female; and
   (c) married or single

   Of these policyholders, 3000 are young, 4600 are male, and 7000 are married. The policyholders can also be classified as 1320 young males, 3010 married males, and 1400 young married persons. Finally, 600 of the policyholders are young married males. Calculate the number of the company's policyholders who are young, female, and single.

WANT $n(\text{YOUNG} \cap \text{FEMALE} \cap \text{SINGLE}) = 10000 - n(\text{OLD} \cup \text{MALE} \cup \text{MARRIED})$

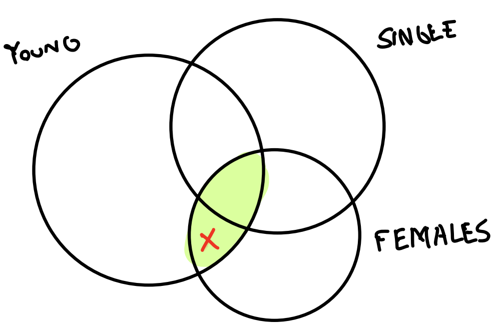
-OR-

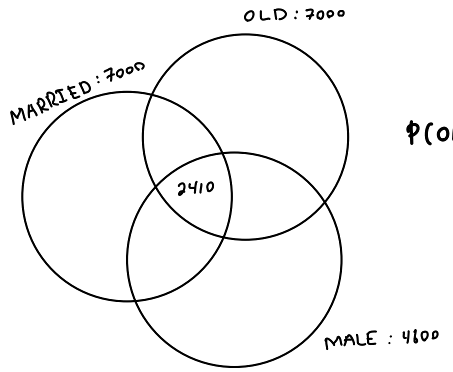
$\underbrace{\text{YOUNG FEMALE}}$: $3000 - 1320 = 1680$

$\underbrace{\text{YOUNG MARRIED FEMALES}}$: $1400 - 600$

$: 800$

$1680 - 800 = \boxed{880}$





$3010 - 600 = 2410$

OLD MARRIED MALES.

$P(\text{OLD} \cup \text{MARRIED} \cup \text{MALE}) =$

$14000 + 4600 +$

$- 3280$ (OLD MALES)

$- 3010$ (MARRIED MALES)

$- 5600$

$+ 2410$

9120

SO $\boxed{10000 - 9120 = 880}$

5. An insurance company estimates that 40% of policyholders who have only an auto policy will renew next year and 60% of policyholders who have only a homeowners policy will renew next year. The company estimates that 80% of policyholders who have both an auto policy and a homeowners policy will renew at least one of those policies next year.

   Company records that that 65% of policyholders have an auto policy, 50% of policyholders have a homeowners policy, and 15% of policyholders have both an auto policy and a homeowners policy.

   Using the company's estimates, calculate the percentage of policyholders that will renew at least one policy next year.

AUTO: 0.65

HOME: 0.5

BOTH: 0.15.

AUTO < 0.4 RENEW / 0.6 DON'T

HOME < 0.6 RENEW / 0.4 DONT

BOTH < 0.8 RENEW / 0.2 DONT

P (RENEW AT LEAST ONE POLICY)

$\underset{\text{NO HOME}}{\text{AUTO \& RENEW}} + \underset{\text{NO AUTO}}{\text{HOME \& RENEW}} + \text{BOTH \& RENEW.}$

$(0.5)(0.4) + (0.35)(0.6) + (0.15)(0.8) = 0.53$

6. Among a large group of patients recovering from shoulder injuries, it is found that 22% visit both a physical therapist and a chiropractor, whereas 12% visit neither of these. The probability that a patient visits a chiropractor exceeds 0.14 the probability that a patient visits a physical therapist.

   Calculate the probability that a randomly chosen member of this group visits a physical therapist.

A: PYSICAL THERAPIST

B: CHIROPRACTOR

$P(A \cap B) = 0.22$

$P(A^c \cap B^c) = 0.12$

$P(A) = ?$

$P(B) = P(A) + 0.14$

$P((A \cup B)^c) = 0.12$

$P(A \cup B) = 0.88$

$P(A \cup B) = P(A) + P(B) - 0.22$

$0.88 = P(A) + P(B) - 0.22$

$1.10 = P(A) + P(A) + 0.14$

$0.96 = 2P(A)$

$\boxed{P(A) = 0.48}$

7. An insurance company examines its pool of auto insurance customers and gathers the following information:

   (a) All customers insure at least one car.
   (b) 70% of the customers insure more than one car.
   (c) 20% of the customers insure a sports car. $(0.7)(0.15)$
   (d) Of those customers who insure more than one car, 15% insure a sports car.

   Calculate the probability that a randomly selected customers insures exactly one car and that car is not a sports car.

A : MORE THAN ONE CAR

B : SPORTS CAR.

C : AT LEAST ONE CAR

$P(A) = 0.7$ $\quad P(A \cap B) = 0.7(0.15) \quad P(A^c \cup B^c) = 0.85$

$P(B) = 0.2$

$$P(A^c \cap B^c) = \underset{0.85}{P(A^c \cup B^c)} - P(A^c) - P(B^c)$$

$$P(A^c \cup B^c) = P(A^c) + P(B^c) - P(A^c \cap B^c)$$

$$P(A^c \cap B^c) = \underset{0.3}{P(A^c)} + \underset{0.8}{P(B^c)} - \underset{0.895}{P(A^c \cup B^c)}$$

$$\boxed{= 0.205}$$

8. An actuary studying the insurance preferences of automobile owners makes the following conclusions:

   (a) An automobile owner is twice as likely to purchase collision coverage as disability coverage.
   (b) The event that an automobile owner purchases collision coverage is independent of the event that he or she purchases disability coverage.
   (c) The probability that an automobile owner purchases both collision and disability collision coverage is 0.15.

   Calculate the probability that an automobile owner purchases neither collision nor disability coverage.

$P(A^c \cup B^c) = 0.85$

A: COLLISION COVERAGE

B : DISABILITY COVERAGE

$P(A) = 2P(B)$

A INDEPENDENT OF B

SO $P(A) \cdot P(B) = P(A \cap B)$

$P(A^c \cap B^c)$

$P(A \cap B) = 0.15$

$(0.15) = 2P(B)^2$

$P(B) = \sqrt{\frac{0.15}{2}}$

$P(B^c) = \left(1 - \sqrt{\frac{0.15}{2}}\right)$

$P(A^c) = \left(1 - 2\sqrt{\frac{0.15}{2}}\right)$

$$P(A^c \cap B^c) = \left(1 - \sqrt{\frac{0.15}{2}}\right)\left(1 - 2\sqrt{\frac{0.15}{2}}\right)$$

$$\boxed{= 0.33}$$

9. An insurance company pays hospital claims. The number of claims that include emergency room operations or operating room charges is 85% of the total number of claims. The number of claims that do not include emergency room charges is 25% of the total number of claims. The occurrence of emergency room charges is independent of the occurrence of operating room charges on hospital claims.

   Calculate the probability that a claim submitted to the insurance company includes operating room charges.

A: EMERGE

B: OPERATINGROOMS

$n(A \cup B) = 0.85n()$

$n(A^c) = 0.25n()$

A INDEPENDENT OF B.

$P(B) = ?$

$P(A \cup B) = 0.85$

$P(A^c) = 0.25$

$P(A) = 0.75$

$P(A \cap B) = P(A) \cdot P(B)$

$n(A) = 0.75n()$

$P(B) = \underbrace{P(A \cup B)}^{0.85} - \overbrace{P(A)}^{0.75} + \underbrace{P(A \cap B)}_{P(A) \cdot P(B)}$

$P(B) = 0.1 + \underbrace{P(A)}_{0.75} \cdot P(B)$

$P(B) - 0.75P(B) = 0.05$

$\boxed{P(B) = \frac{0.10}{0.25} = \frac{2}{5}}$

10. An insurance agent's files reveal the following facts about his policyholders:

   (a) 243 own auto insurance.
   (b) 207 own homeowner insurance.
   (c) 55 own life insurance and homeowner insurance.
   (d) 96 own auto insurance and homeowner insurance.
   (e) 32 own life insurance, auto insurance, and homeowner insurance.
   (f) 76 more clients own only auto insurance than only life insurance.
   (g) 270 own only one of these three insurance products

   Calculate the total number of the agent's policyholders who own at least one of these three insurance products.

A: AUTO

B: HOMEOWERS

C: LIFE

✓ $n(A) = 243$

✓ $n(B) = 207$

✓ $n(B \cap C) = 55$

✓ $n(A \cap B) = 96$

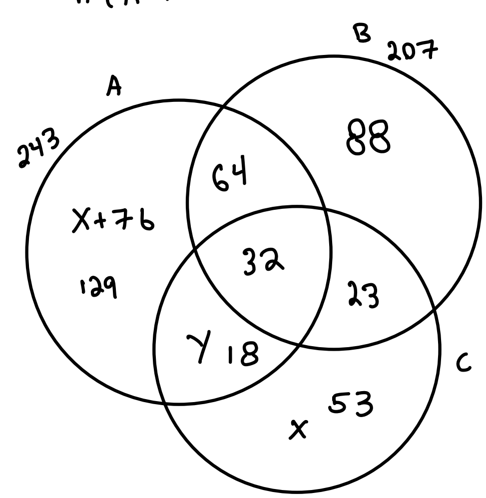
✓ $n(A \cap B \cap C) = 32$

$n(A) = n(C) + 76$

ONLY ONE : 270

$n(A \cap B^c \cap C^c) + n(B \cap C^c \cap A^c) + n(C \cap B^c \cap A^c)$

$n(A^c \cap B^c \cap C^c)$



$Y = 243 - 64 - 32 - 129$

$= 18$

$x + (x+76) + 88 = 270$

THUS

$x = 53$

TOTAL = ADD THEM ALL TOGETHER

$\boxed{= 407}$

11. A profile of the investments owned by an agents clients follows:

(a) 228 own annuities.
(b) 220 own mutual funds.
(c) 98 own life insurance and mutual funds.
(d) 93 own annuities and mutual funds.
(e) 16 own annuities, mutual funds, and life insurance.
(f) 45 more clients own only life insurance than own only annuities.
(g) 290 own only one type of investment (i.e. annuity, mutual fund, or life insurance).

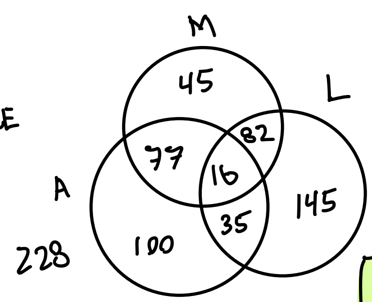
WANT n().

159 + 16

A: Annuities

M: MUTUAL FUNDS

L: LIFE INSURANCE



$n(A) = 228$

$n(M) = 220$

$n(M \cap L) = 98$

$n(A \cap M) = 93$

$n(A \cap M \cap L) = 16$

$\boxed{\text{ADDING GIVES } 500}$

$220 - (77 + 82 + 16)$

$= 220 - 175$

$= 45$

$2x + 90 = 290$

$x = 100$

12. An actuary compiles the following information from a portfolio of 1000 homeowners insurance policies.

(a) 130 policies insure three-bedroom homes.
(b) 280 policies insure one-story homes.
(c) 150 policies insure two-bath homes.
(d) 30 policies insure three-bedroom, two-bath homes.
(e) 50 policies insure one-story, two-bath homes.
(f) 40 policies insure three-bedroom, one-story homes.
(g) 10 policies insure three-bedroom, one-story, two-bath homes.

Calculate the number of homeowners policies in the portfolio that insure neither one-story nor two-bath nor three-bedroom homes.

A : THREE BEDROOM

B : ONE-STORY

C : TWO BATH

$n(A \cup B \cup C) = 130 + 280 + 150 - 30 - 50 - 40 + 10 = 450$

$n(A) = 130$

$n(B) = 280$

$n(C) = 150$

$n(A \cap C) = 30$

$n(B \cap C) = 50$

$n(A \cap B) = 40$

$n(A \cap B \cap C) = 10$

WANT: $n(A^c \cap B^c \cap C^c)$

$n((A \cup B \cup C)^c) = 1000 - 450 = \boxed{550}$

13. A company sells two types of life insurance policies (P and Q) and one type of health insurance policy. A survey of potential customers revealed the following:

    (a) No survey participant wanted to purchase both life policies.
    (b) Twice as many survey participants wanted to purchase life policy P as life policy Q.
    (c) 45% of survey participants wanted to purchase the health policy.
    (d) 18% of survey participants wanted to purchase only the health policy.
    (e) The event that a survey participant wanted to purchase the health policy was independent of the event that a survey participant wanted to purchase a life policy.

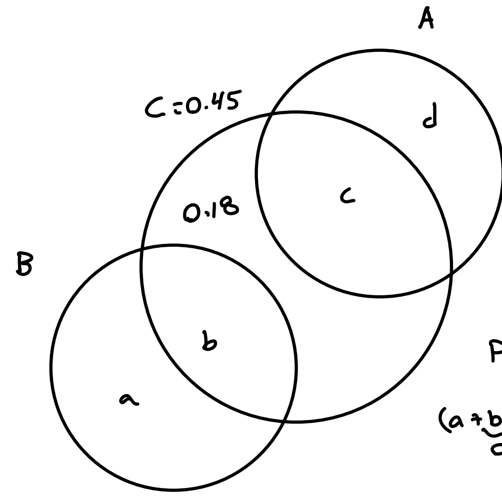
    Calculate the probability that a randomly selected participant wanted to purchase exactly one policy.

A: POLICY P

B: POLICY Q

C: HEALTH.

$n(A \cap B) = 0$



$a + b = 2(c + d)$

$b + c + 0.18 = 0.45$

$b + c = 0.27$

$P((A \cup B) \cap C) = P(A \cup B) \cdot P(C)$

$(a + \underbrace{b + c}_{0.27} + d)(0.45) = 0.27$

$a + d = 0.33$

$\boxed{0.33 + 0.18 = 0.51}$

14. In a study of driver safety, drivers were categorized according to three risk factors. Exactly 1000 drivers exhibited each individual risk factor. Also, for each of the risk factors, there were exactly 400 drivers exhibiting that risk factor and neither of the other two risk factors. Finally, there were exactly 300 drivers who exhibited all three risk factors and 500 who exhibited none of the three risk factors.

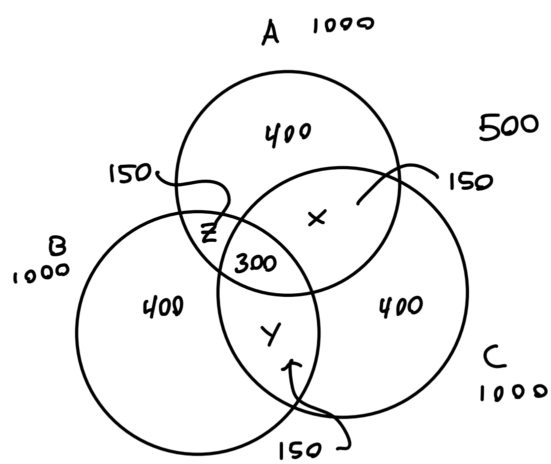
    Calculate the number of drivers in the study.

A

B

C

$n(A) = 1000$



$x + z = 300$

$z + y = 300$

$x + y = 300$

$z - y = 0$

$z + y = 300$

$2z = 300$

$z = 150$

$\boxed{400 + 600 + 400 + 400 + 150 + 500 = 2450}$

15. An insurance company examines its pool of auto insurance customers and gathers the following information:

EXACTLY THE SAME AS PREVIOUS QUESTION!

(a) All customers insure at least one car.
(b) 64% of the customers insure more than one car.
(c) 20% of the customers insure a sports car.
(d) Of those customers who insure more than one car, 15% insure a sports car.

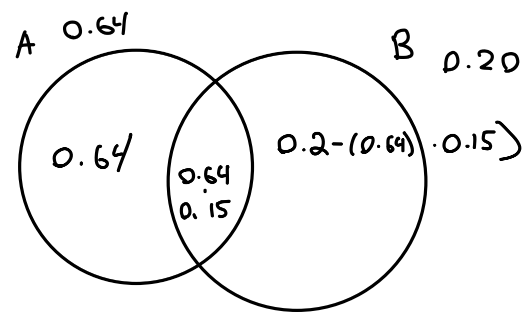
Calculate the probability that a randomly selected customer insures exactly one car, and that the car is not a sports car.

A: 0.64: MORE THAN ONE CAR.

B: 0.20 : SPORTS CAR

$(A \cap B) = (0.64)(0.15)$

$=$



$P(A^c \cap B^c) = 1 - P(A \cup B)$

$= 1 - (0.64 + 0.2 - (0.64)(0.15))$

$\boxed{= 0.256}$

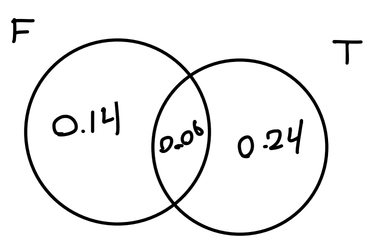
16. In any 12-month period, the probability that a home is damaged by fire is 20% and the probability of a theft loss at a home is 30%. The occurrences of fire damage and theft loss are independent events. Calculate the probability that a randomly selected home will either be damaged by fire or will have a theft loss, but not both, during the next year.

F : 0.2

T : 0.3

$P(F \cap T) = P(F)P(T)$

$= (0.2)(0.3) = 0.06$



$\boxed{0.38}$