Sampling Distribution of $\bar{x}_1 - \bar{x}_2$

AP Statistics · Mr. Merrick · January 22, 2026

Suppose we take two random samples from two populations.

- Sample 1 has size n_1, population mean μ_1, population standard deviation σ_1, and sample mean $\bar{x}_1$.
- Sample 2 has size n_2, population mean μ_2, population standard deviation σ_2, and sample mean $\bar{x}_2$.

We are interested in the sampling distribution of the difference

$$\bar{x}_1 - \bar{x}_2,$$

which is a random variable because both samples are **random**.
To determine whether $\bar{x}_1 - \bar{x}_2$ can be modeled using a normal distribution, we must check conditions related to **independence** and **normality / large sample size**.

Independence Condition	Normality / Large Sample Condition
• The two samples are independent of each other • Each sample is taken randomly • If sampling without replacement: $n_1 \leq 0.10N_1 \quad \text{and} \quad n_2 \leq 0.10N_2$	• Both populations are normal, **or** • $n_1 \geq 30$ and $n_2 \geq 30$

If all of the above conditions are satisfied, then

$$\bar{x}_1 - \bar{x}_2 \approx \mathcal{N}\left(\mu_1 - \mu_2,\ \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right).$$

1. For each situation below, determine whether the sampling distribution of $\bar{x}_1 - \bar{x}_2$ can be modeled using the **normal model**:

$$\bar{x}_1 - \bar{x}_2 \approx \mathcal{N}\left(\mu_1 - \mu_2,\ \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right).$$

Justify your answer by referring to the appropriate conditions.

(a) A researcher compares average commute times in two large cities. A random sample of $n_1 = 45$ commuters is selected from City A and $n_2 = 50$ commuters from City B. The distributions of commute times in both cities are strongly right-skewed.

(b) A school compares test scores between two small classes. A random sample of $n_1 = 12$ students is taken from Class A and $n_2 = 10$ students from Class B. Both populations are approximately normal.

(c) A company compares assembly times for two machines. A random sample of $n_1 = 18$ items from Machine A and $n_2 = 22$ items from Machine B is selected. Both distributions are strongly skewed.

(d) A teacher compares quiz scores between two classes by sampling $n_1 = 14$ students from a class of $N_1 = 25$ and $n_2 = 16$ students from a class of $N_2 = 30$.

2. A manufacturer compares the average lifetime of batteries produced by two factories.

- Factory A: $\mu_1 = 1200$ hours, $\sigma_1 = 150$ hours, $n_1 = 64$
- Factory B: $\mu_2 = 1150$ hours, $\sigma_2 = 160$ hours, $n_2 = 81$

Let $\bar{x}_1 - \bar{x}_2$ represent the difference in sample mean lifetimes (Factory A minus Factory B).

(a) Explain why a normal model for $\bar{x}_1 - \bar{x}_2$ is appropriate.

(b) Find the probability that the difference in sample means is greater than 75 hours.

3. A college compares the average number of hours students study per week between freshmen and seniors.

- Freshmen: $\mu_1 = 11$ hours, $\sigma_1 = 3.5$ hours, $n_1 = 100$
- Seniors: $\mu_2 = 13$ hours, $\sigma_2 = 4.0$ hours, $n_2 = 90$

Let $\bar{x}_1 - \bar{x}_2$ represent the difference in sample means (freshmen minus seniors).

(a) Explain why a normal model for $\bar{x}_1 - \bar{x}_2$ is appropriate.

(b) Find the probability that the freshmen sample mean is less than or equal to the senior sample mean. That is, find $P(\bar{x}_1 - \bar{x}_2 \leq 0)$.