Sampling Distribution of $\hat{p}_1 - \hat{p}_2$

AP Statistics · Mr. Merrick · January 22, 2026

Suppose we take two random samples from two populations.

- Sample 1 has size n_1 with true proportion p_1 and sample proportion

$$\hat{p}_1 = \frac{X_1}{n_1}.$$

- Sample 2 has size n_2 with true proportion p_2 and sample proportion

$$\hat{p}_2 = \frac{X_2}{n_2}.$$

We are interested in the sampling distribution of the difference

$$\hat{p}_1 - \hat{p}_2,$$

which is a random variable because both samples are **random**. To determine whether $\hat{p}_1 - \hat{p}_2$ can be modeled using a normal distribution, we must check conditions related to **independence** and **normality**.

Independence Condition	**Normality (Success–Failure) Condition**
• The two samples are independent of each other • Each sample is taken randomly • If sampling without replacement: $n_1 \le 0.10N_1$ and $n_2 \le 0.10N_2$	• For Sample 1: $n_1p_1 \ge 10$ and $n_1(1-p_1) \ge 10$ • For Sample 2: $n_2p_2 \ge 10$ and $n_2(1-p_2) \ge 10$

If all of the above conditions are satisfied, then

$$\hat{p}_1 - \hat{p}_2 \approx \mathcal{N}\left(p_1 - p_2,\ \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}\right).$$

1. For each situation below, determine whether the sampling distribution of $\hat{p}_1 - \hat{p}_2$ can be modeled using the **normal distribution**:

$$\hat{p}_1 - \hat{p}_2 \approx \mathcal{N}\left(p_1 - p_2,\ \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}\right).$$

Justify your answer by referring to the appropriate conditions.

(a) A school compares the proportion of students who play a sport between freshmen and seniors. A random sample of $n_1 = 40$ freshmen and $n_2 = 35$ seniors is taken. The true participation rates are $p_1 = 0.10$ and $p_2 = 0.20$.

(b) A quality-control inspector compares defect rates from two machines. Sample 1 includes $n_1 = 120$ items and Sample 2 includes $n_2 = 150$ items. The true defect rates are $p_1 = 0.18$ and $p_2 = 0.22$.

(c) A teacher compares homework completion rates between two classes of sizes $N_1 = 28$ and $N_2 = 30$ by sampling $n_1 = 15$ and $n_2 = 18$ students without replacement.

2. A news organization compares support for a proposed law in two different states.

- In State A, the true proportion of adults who support the law is $p_1 = 0.52$. A random sample of $n_1 = 200$ adults is selected.
- In State B, the true proportion of adults who support the law is $p_2 = 0.45$. A random sample of $n_2 = 180$ adults is selected.

Let $\hat{p}_1$ and $\hat{p}_2$ be the sample proportions from State A and State B, respectively.

(a) Explain why the sampling distribution of $\hat{p}_1 - \hat{p}_2$ can be approximated by a normal distribution.

(b) Find the probability that the difference in sample proportions is *greater than* 0.12. That is, find $P(\hat{p}_1 - \hat{p}_2 > 0.12)$.

3. A school district compares sleep habits between middle school students and high school students.

- Among middle school students, the true proportion who get at least 8 hours of sleep on school nights is $p_1 = 0.30$. A random sample of $n_1 = 150$ students is selected.
- Among high school students, the true proportion is $p_2 = 0.25$. A random sample of $n_2 = 120$ students is selected.

Let $\hat{p}_1 - \hat{p}_2$ represent the difference in sample proportions (middle school minus high school).

(a) Explain why a normal model for $\hat{p}_1 - \hat{p}_2$ is appropriate.

(b) Find the probability that the middle school sample proportion is *less than or equal to* the high school sample proportion. That is, find $P(\hat{p}_1 - \hat{p}_2 \leq 0)$.