Sampling Distributions – Extra Practice

AP Statistics · Sampling Distributions · Mr. Merrick · January 29, 2026

1. Suppose it is known that 43% of Americans own an iPhone. A random sample of 50 Americans is selected.

   (a) What is the probability that the sample proportion who own an iPhone is between 45% and 50%?

   Solution: Let $p = 0.43$ and $n = 50$.

   Conditions:

   - Random sample stated.
   - Independence: n is less than 10% of the population.
   - Normal: $np = 21.5$ and $n(1-p) = 28.5$.

   $$\sigma_{\hat{p}} = \sqrt{\frac{(0.43)(0.57)}{50}} \approx 0.0699$$

   $$P(0.45 < \hat{p} < 0.50) = P(0.29 < Z < 1.00) \approx 0.2289$$

   (b) If a random sample of 75 Americans is selected, what is the probability that more than 50% own an iPhone?

   Solution: Conditions are satisfied: random, independent, and np and $n(1-p)$ are both greater than 10.

   $$\sigma_{\hat{p}} = \sqrt{\frac{(0.43)(0.57)}{75}} \approx 0.0572$$

   $$P(\hat{p} > 0.50) = P\left(Z > \frac{0.50 - 0.43}{0.0572}\right) = P(Z > 1.22) \approx 0.1104$$

2. Which of the following statements is incorrect?

   A. The larger the sample size, the larger the spread of the sampling distribution.

   B. Provided the population is much larger than the sample, the spread does not depend on population size.

   C. Bias affects the center, not the spread, of a sampling distribution.

   D. A sample distribution and a sampling distribution describe different concepts.

   E. As sample size increases, the sample distribution more closely resembles the population distribution.

   Solution: The first statement is incorrect. Increasing sample size decreases the spread of the sampling distribution.

3. Which of the following statements is incorrect?

   A. For a random sample, the sampling distribution of $\bar{x}$ has mean μ.

   B. When observations are independent, the standard deviation of $\bar{x}$ is $\sigma/\sqrt{n}$.

   C. If the population distribution is normal, then $\bar{x}$ is normally distributed for any sample size n.

   D. For sufficiently large n, the sampling distribution of $\bar{x}$ is approximately normal, even if the population distribution is not normal.

   E. The Central Limit Theorem applies even when observations are not independent.

   Solution: The incorrect statement is the last one. Independence is a required condition for the Central Limit Theorem. Without independence, the sampling distribution of $\bar{x}$ may not be approximately normal, even for large sample sizes.

4. Which of the following is a true statement?

   A. The mean of $\hat{p}$ differs from p by about 1.96 standard deviations.

   B. The standard deviation of $\hat{p}$ is $\sqrt{np(1-p)}$.

   C. $\hat{p}$ is normal whenever $n \geq 30$.

   D. The sample proportion is a random variable with a probability distribution.

   E. All of the above are true.

   Solution: The sample proportion varies from sample to sample, making it a random variable.

5. Which of the following statements is incorrect?

   A. Sample statistics estimate population parameters.

   B. Smaller samples tend to have more variability.

   C. Parameters are fixed; statistics vary.

   D. The sample distribution becomes normal as n increases.

   E. All of the above are true.

   Solution: Sample size affects the sampling distribution, not the population distribution.

6. The price of a dozen donuts is normally distributed with mean \$10.00 and standard deviation \$0.50. A customer buys a dozen donuts on each of five days. What is the probability the total cost exceeds \$52.00?

Solution: Let T be the total cost over five days.

Conditions:

- Each day's price is normally distributed.
- The prices on different days are independent.

Because the sum of independent normal random variables is also normally distributed, T is normally distributed.

$$\mu_T = 5(10) = 50, \quad \sigma_T = \sqrt{5}(0.50) \approx 1.118.$$

$$P(T > 52) = P\left(Z > \frac{52 - 50}{1.118}\right) = P(Z > 1.79) \approx 0.0368.$$

7. Suppose 15% of mines in a region strike gold. In a random sample of 200 mines, what is the probability that more than 18% strike gold?

Solution: Let $\hat{p}$ be the sample proportion of mines that strike gold, where $p = 0.15$ and $n = 200$.

Conditions:

- **Random:** The sample is stated to be random.
- **Independence:** The sample size of 200 mines is less than 10% of all mines in the region.
- **Normal:** $np = 30$ and $n(1 - p) = 170$, both at least 10.

Therefore, the sampling distribution of $\hat{p}$ is approximately normal with

$$\sigma_{\hat{p}} = \sqrt{\frac{(0.15)(0.85)}{200}} \approx 0.02525.$$

$$P(\hat{p} > 0.18) = P\left(Z > \frac{0.18 - 0.15}{0.02525}\right) = P(Z > 1.19) \approx 0.1174.$$

8. In an SRS of 1000 people, 6.7% graduate with a STEM degree. What is the probability that more than 8% graduate with a STEM degree?

Solution: Let $\hat{p}$ be the sample proportion of people who graduate with a STEM degree, where $p = 0.067$ and $n = 1000$.

Conditions:

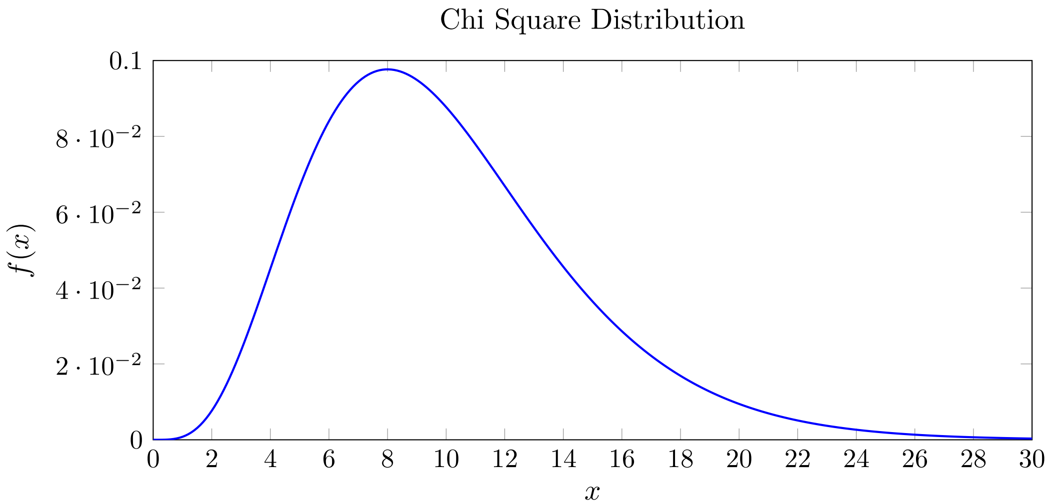
- **Random:** The sample is an SRS.
- **Independence:** The sample size is less than 10% of the population.
- **Normal:** $np = 67$ and $n(1 - p) = 933$, both greater than 10.

Thus, the sampling distribution of $\hat{p}$ is approximately normal with

$$\sigma_{\hat{p}} = \sqrt{\frac{(0.067)(0.933)}{1000}} \approx 0.00791.$$

$$P(\hat{p} > 0.08) = P\left(Z > \frac{0.08 - 0.067}{0.00791}\right) = P(Z > 1.64) \approx 0.0501.$$

9. $X_1, \ldots, X_{500}$ are independent random variables with $X_i \sim \chi^2_{10}$, where $E(X) = 10$ and $\text{Var}(X) = 20$.



Describe the sampling distribution of $\bar{x}$.

Solution: Because the sample size is large ($n = 500$) and the observations are independent, the Central Limit Theorem applies, even though the population distribution is skewed.

The sampling distribution of $\bar{x}$ has

$$\mu_{\bar{x}} = 10 \quad \text{and} \quad \sigma_{\bar{x}} = \sqrt{\frac{20}{500}} = 0.20.$$

Therefore,

$$\bar{x} \sim \text{Normal}\,(10, 0.20)\,.$$

10. In each of the following cases, state whether it is appropriate to use

$$\frac{\bar{x} - \mu}{s/\sqrt{n}} \sim t_{n-1}.$$

(a) $n = 100$, population is Normal$(10, 2)$.

Solution: Appropriate. The population distribution is normal and the sample size is large.

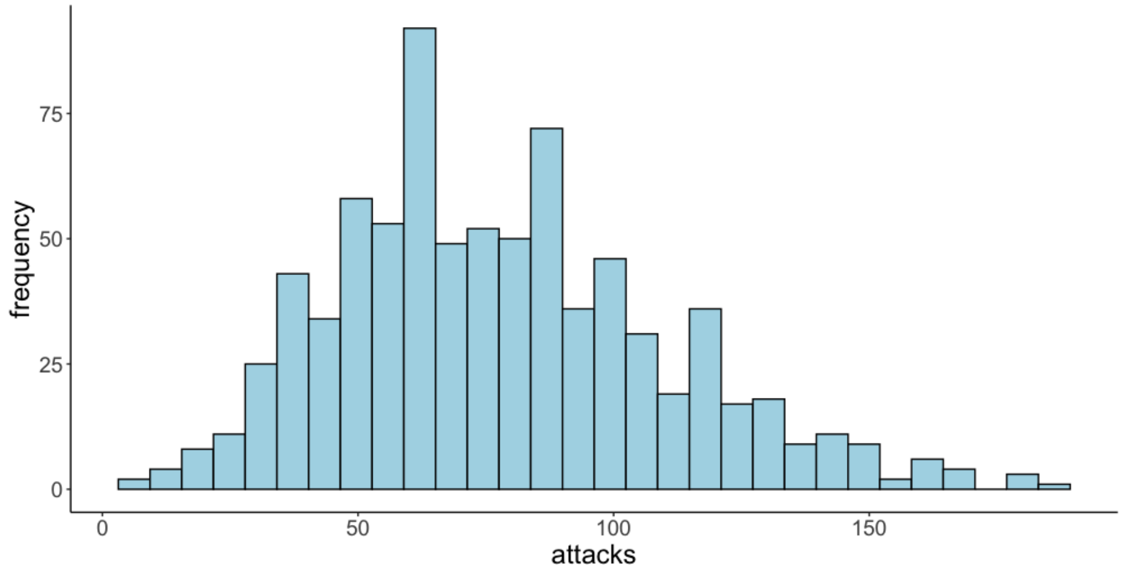
(b) $n = 10$, population is Normal$(10, 2)$.

Solution: Appropriate. Although the sample size is small, the population distribution is normal.

(c) $n = 10$, population is Exponential(3).

Solution: Not appropriate. The population distribution is strongly skewed and the sample size is too small for the Central Limit Theorem to apply.

11. A random sample of $n = 801$ Pokémon has $\bar{x} = 78$ and $s = 32$. The true mean attack score is $\mu = 70$.



What is the probability that a future sample has an average attack score less than 75?

Solution: Because the sample size is very large, the sampling distribution of $\bar{x}$ is approximately normal.

$$\sigma_{\bar{x}} = \frac{32}{\sqrt{801}} \approx 1.1307$$

$$P(\bar{x} < 75) = P\left(Z < \frac{75 - 70}{1.1307}\right) = P(Z < 4.42) \approx 0.99999.$$

12. A factory produces bolts with mean length 5.0 cm and standard deviation 0.4 cm. A random sample of 64 bolts is selected. What is the probability the sample mean is between 4.9 cm and 5.1 cm?

Solution: Let $\bar{x}$ be the sample mean bolt length.

Conditions:

- **Random:** The problem states a random sample.
- **Independence:** The sample size of 64 bolts is less than 10% of the total production.
- **Normal:** The sample size is large ($n = 64$), so by the Central Limit Theorem, the sampling distribution of $\bar{x}$ is approximately normal.

The sampling distribution of $\bar{x}$ has

$$\mu_{\bar{x}} = 5.0 \quad \text{and} \quad \sigma_{\bar{x}} = \frac{0.4}{\sqrt{64}} = 0.05.$$

$$P(4.9 < \bar{x} < 5.1) = P(-2 < Z < 2) \approx 0.9545.$$

13. In a population, 62% of voters support a ballot initiative. A random sample of 400 voters is selected. What is the probability the sample proportion is within 3 percentage points of the true proportion?

Solution: Let $\hat{p}$ be the sample proportion of voters who support the initiative, where $p = 0.62$ and $n = 400$.

Conditions:

- **Random:** The sample is stated to be random.
- **Independence:** The sample size is less than 10% of the population.
- **Normal:** $np = 248$ and $n(1 - p) = 152$, both greater than 10.

Therefore, the sampling distribution of $\hat{p}$ is approximately normal with

$$\sigma_{\hat{p}} = \sqrt{\frac{(0.62)(0.38)}{400}} \approx 0.02427.$$

$$P(|\hat{p} - 0.62| < 0.03) = P\left(|Z| < \frac{0.03}{0.02427}\right) = P(|Z| < 1.24) \approx 0.784.$$

14. The weights of cereal boxes have mean 18 oz and standard deviation 0.6 oz. What sample size is required so that the probability the sample mean is within 0.1 oz of the true mean is at least 0.95?

Solution: Let $\bar{x}$ be the sample mean box weight.

Conditions:

- The population standard deviation $\sigma = 0.6$ oz is known.
- The desired probability statement assumes the sampling distribution of $\bar{x}$ is approximately normal. This is reasonable for sufficiently large n by the Central Limit Theorem.

To be within 0.1 oz of the true mean with probability at least 0.95,

$$P(|\bar{x} - \mu| < 0.1) = 0.95.$$

Using the 95% rule for a normal distribution:

$$1.96\left(\frac{0.6}{\sqrt{n}}\right) \leq 0.1.$$

Solving,

$$n \geq \left(\frac{1.96(0.6)}{0.1}\right)^2 \approx 138.3.$$

Therefore, a sample size of at least $\boxed{139}$ boxes is required.