When Parameters Are Unknown

AP Statistics · Mr. Merrick · January 21, 2026

In practice, we almost never know the true values of population parameters.

- The population proportion p is unknown.
- The population standard deviation σ is unknown.

However, probability models for sampling distributions rely on these parameters. To continue making probability statements, we must replace unknown parameters with estimates from the sample. This leads us to the idea of a standard error, which estimates the standard deviation of a sampling distribution using sample data.

All sampling distributions in this document require:

- random sampling from the population
- independence:
  - observations are independent, and
  - if sampling without replacement, $n \leq 0.10N$.

One-Sample Proportion ($\hat{p}$)

Suppose we take a random sample of size n from a population with unknown proportion p.

- The statistic $\hat{p}$ estimates the unknown parameter p.
- The true standard deviation of $\hat{p}$ depends on p, which is unknown.

Parameter Known (Theoretical)	Parameter Unknown (Practical)
$\text{SD}(\hat{p}) = \sqrt{\dfrac{p(1-p)}{n}}$	$\text{SE}(\hat{p}) = \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

Normality (success–failure) condition:

$$n\hat{p} \geq 10 \quad \text{and} \quad n(1-\hat{p}) \geq 10.$$

If the random, independence, and success–failure conditions are satisfied, then

$$\hat{p} \approx \mathcal{N}\left(p,\ \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right),$$

and probability calculations use the standard normal (z) distribution.

Example:

A random sample of $n = 150$ voters is selected from a large population. In the sample, 84 voters support a candidate. Find the probability that $\hat{p} > 0.60$.

Solution.

$$\hat{p} = \frac{84}{150} = 0.56.$$

The sample is random, and the population is large enough to satisfy the 10% condition. The success–failure condition is met:

$$150(0.56) = 84 \geq 10, \quad 150(0.44) = 66 \geq 10.$$

Standard error:

$$\text{SE} = \sqrt{\frac{0.56(0.44)}{150}} \approx 0.0406.$$

$$z = \frac{0.60 - 0.56}{0.0406} \approx 0.99.$$

$$P(\hat{p} > 0.60) \approx P(Z > 0.99) \approx 0.161.$$

One-Sample Mean ($\overline{x}$)

Suppose we take a random sample of size n from a population with unknown mean μ and unknown standard deviation σ.

- The statistic $\overline{x}$ estimates μ.
- The sample standard deviation s estimates σ.

Parameter Known (Theoretical)	Parameter Unknown (Practical)
$\text{SD}(\overline{x}) = \frac{\sigma}{\sqrt{n}}$	$\text{SE}(\overline{x}) = \frac{s}{\sqrt{n}}$

Normality / large sample condition:

- the population distribution is normal, or
- the sample size satisfies $n \geq 30$.

If the random, independence, and normality/large-sample conditions are satisfied, then

$$\overline{x} \approx t_{n-1}\left(\mu,\ \frac{s}{\sqrt{n}}\right),$$

and probability calculations use a t distribution with $n - 1$ degrees of freedom.

Example (Mean):

A random sample of $n = 25$ students has a mean test score of 78 with a sample standard deviation of $s = 6$. Assuming the population distribution is approximately normal, find the probability that $\overline{x} > 80$.

Solution. The sample is random and represents less than 10% of the population. The population distribution is approximately normal, so a t model is appropriate.
Standard error:

$$\mathrm{SE} = \frac{6}{\sqrt{25}} = 1.2.$$

$$t = \frac{80 - 78}{1.2} \approx 1.67.$$

Using a t distribution with 24 degrees of freedom,

$$P(\overline{x} > 80) \approx P(T_{24} > 1.67) \approx 0.055.$$

Summary

Statistic	Standard Error	Reference Distribution
$\hat{p}$	$\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	Normal (z)
$\overline{x}$	$\frac{s}{\sqrt{n}}$	t with $n-1$ df

These sampling distributions provide the foundation for confidence intervals and significance tests developed in the inference units.