Sample Size for Estimating a Population Proportion

AP Statistics · Mr. Merrick · February 3, 2026

When planning a confidence interval for a population proportion p, we often want to know how large a sample is needed to guarantee a desired margin of error.

Margin of error for a proportion

For a one-sample z interval,

$$\hat{p} \pm z^{\star}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

The margin of error is

$$\mathrm{ME} = z^{\star}\sqrt{\frac{p(1-p)}{n}}$$

where p is a planning value for the population proportion. To guarantee a margin of error no larger than a chosen value ME, solve for n:

$$\mathrm{ME} = z^{\star}\sqrt{\frac{p(1-p)}{n}} \Rightarrow n = \frac{(z^{\star})^2\, p(1-p)}{\mathrm{ME}^2}$$

Always round up to the nearest whole number.

Why $p = 0.50$ is the most conservative choice

If no prior estimate of p is available, we choose a value that makes the required sample size n as large as possible. This occurs when $p(1-p)$ is maximized. Let

$$f(p) = p(1-p), \quad 0 \le p \le 1.$$

$$f'(p) = 1 - 2p.$$

Set the derivative equal to zero:

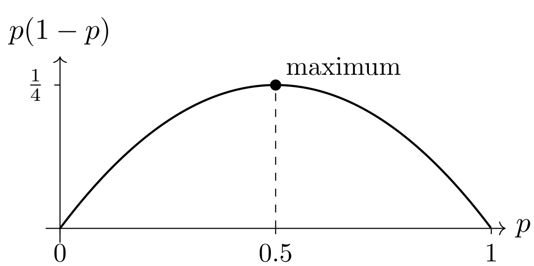
$$1 - 2p = 0 \quad \Rightarrow \quad p = \tfrac{1}{2}.$$

The second derivative is

$$f''(p) = -2 < 0,$$

so this critical point is a maximum. Therefore,

$$\max_{0\le p\le 1} p(1-p) = \frac{1}{4}.$$



Using $p = 0.50$ produces the largest possible margin of error and guarantees the sample size will be large enough regardless of the true value of p.

Practice

1. No prior estimate

A school wants to estimate the proportion of students with part-time jobs. They want a 95% confidence interval with ME ≤ 0.03. Find the required sample size.

$$n = \frac{(1.96)^2(0.25)}{(0.03)^2} = \frac{0.9604}{0.0009} = 1067.11\ldots \Rightarrow n = 1068$$

2. Using a prior estimate

A previous survey suggests $p \approx 0.32$. Find the required sample size for a 90% confidence interval with ME ≤ 0.04.

$$n = \frac{(1.645)^2(0.32)(0.68)}{(0.04)^2} = \frac{0.58883}{0.0016} = 368.02\ldots \Rightarrow n = 369$$

3. Comparing conservative vs. informed planning

A city wants a 99% confidence interval with ME ≤ 0.02.

1. Find n using no prior estimate.
2. Find n assuming $p \approx 0.70$.

$$n_{\text{conservative}} = \frac{(2.576)^2(0.25)}{(0.02)^2} = \frac{1.658944}{0.0004} = 4147.36 \Rightarrow n = 4148$$

$$n_{p=0.70} = \frac{(2.576)^2(0.21)}{(0.02)^2} = \frac{1.393513}{0.0004} = 3483.78 \Rightarrow n = 3484$$