One Sample Confidence Intervals – Extra Practice

AP Statistics · Unit 6 · Mr. Merrick · February 2, 2026

1. A school administrator is concerned about students who report being absent due to illness but are later seen attending school-sponsored events on the same day.

   A random sample of 120 illness-related absences was selected, and 31 of those students attended a school event that day.

   (a) Construct and interpret a 95% confidence interval for the proportion of all illness-related absences that are not legitimate.

   Solution: Step 1 — State important information, and define the target
   Let p be the true proportion of all illness-related absences that are not legitimate. We will construct a 95% confidence interval for p.
   $\hat{p} = \dfrac{31}{120} = 0.2583, \quad n = 120, \quad z^\star = 1.96$
   Step 2 — Justify the method (checking conditions)
   Random: A random sample is stated.
   Independence: $n = 120$ is less than 10% of all illness-related absences.
   Normal / Large Sample: $n\hat{p} = 31 \geq 10$ and $n(1-\hat{p}) = 89 \geq 10$.
   Step 3 — Carry out the procedure (computation)

   $$\begin{aligned} \hat{p} \pm z^\star\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} &= 0.2583 \pm 1.96\sqrt{\frac{(0.2583)(0.7417)}{120}} \\ &= 0.2583 \pm 0.0783 \Rightarrow (0.1800,\ 0.3367) \end{aligned}$$

   Step 4 — Interpret the result (meaning in context)
   We are 95% confident that the true proportion of illness-related absences that are not legitimate is between 0.180 and 0.337.

   (b) The administrator estimates that each illegitimate absence costs the school \$42 in lost funding. If the school records 4,800 illness-related absences per year, estimate the annual cost using the confidence interval from part (a).

   Solution: Use the interval from part (a) and apply it to the yearly total.
   Estimated number of illegitimate absences:

   $$4800(0.1800) = 864 \quad \text{to} \quad 4800(0.3367) \approx 1616$$

   Estimated annual cost:

   $$864(42) = \$36{,}288 \quad \text{to} \quad 1616(42) = \$67{,}872$$

   A reasonable 95% interval for the annual financial loss is \$36,288 to \$67,872.

2. A national polling agency surveyed a random sample of 1,250 adults and asked which of the following best reflects their opinion.

Response	Online Shopping	In-store Shopping	No Preference
Percent of sample	46%	41%	13%

(a) Construct and interpret a 99% confidence interval for the proportion of all U.S. adults who prefer shopping in stores.

Solution: Step 1 — State important information, and define the target
Let p be the true proportion of all U.S. adults who prefer shopping in stores. We will construct a 99% confidence interval for p.

$\hat{p} = 0.41, \quad n = 1250, \quad z^\star = 2.576$

Step 2 — Justify the method (checking conditions)
Random: A random sample is stated.
Independence: $n = 1250$ is less than 10% of the population of U.S. adults.
Normal / Large Sample: $n\hat{p} = 1250(0.41) = 512.5 \geq 10$ and $n(1-\hat{p}) = 1250(0.59) = 737.5 \geq 10$.

Step 3 — Carry out the procedure (computation)

$$\hat{p} \pm z^\star \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.41 \pm 2.576\sqrt{\frac{(0.41)(0.59)}{1250}} \approx 0.41 \pm 0.036$$

$$(0.374,\ 0.446)$$

Step 4 — Interpret the result (meaning in context)
We are 99% confident that the true proportion of all U.S. adults who prefer shopping in stores is between 0.374 and 0.446.

(b) One condition for inference is that both $n\hat{p}$ and $n(1-\hat{p})$ are at least 10. Explain why this condition is necessary.

Solution: This condition helps ensure the sampling distribution of $\hat{p}$ is approximately normal. With at least 10 expected successes and 10 expected failures, the normal approximation is accurate enough for the confidence interval to be reliable.

(c) A student suggests using a two-sample z-interval to compare the proportions who prefer online shopping and in-store shopping. Is this appropriate? Justify your answer.

Solution: No. The online and in-store preferences come from the same individuals in one sample, so the two proportions are not from independent samples. A two-sample z-interval for $p_1 - p_2$ requires two independent random samples (or random assignment to two groups).

3. A city council wants to estimate the proportion of residents who support a proposed zoning change.

   (a) What minimum sample size is required to estimate the true proportion with a margin of error no greater than 0.03 at the 95% confidence level? Assume no prior estimate of the proportion is available.

   Solution: Step 1 — State important information, and define the target
   We want a sample size n so that the margin of error for a 95% confidence interval for p is at most 0.03. With no prior estimate, use $\hat{p} = 0.50$ (most conservative).

   Confidence level: 95% so $z^\star = 1.96$.

   Step 2 — Justify the method (setup)
   For a one-proportion confidence interval, the margin of error is

   $$ME = z^\star \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$

   We require $ME \leq 0.03$.

   Step 3 — Carry out the procedure (solve for n)

   $$1.96\sqrt{\frac{(0.50)(0.50)}{n}} \leq 0.03$$

   $$\sqrt{\frac{0.25}{n}} \leq \frac{0.03}{1.96} \quad \Rightarrow \quad \frac{0.25}{n} \leq \left(\frac{0.03}{1.96}\right)^2$$

   $$n \geq \left(\frac{1.96(0.50)}{0.03}\right)^2 = 1067.11$$

   Step 4 — Interpret the result (final answer)
   Round up to guarantee the margin of error: a minimum sample size of $\boxed{1068}$ residents is required.

   (b) Explain why using $p = 0.5$ produces the most conservative sample size estimate.

   Solution: The expression $p(1-p)$ is largest when $p = 0.5$, which makes the standard error (and margin of error) as large as possible for a given n. Using $p = 0.5$ guarantees the computed n will be large enough even if the true proportion is different.