

ARITHMETIC SERIES – FINDING THE SUM

Mr. Merrick · January 27, 2026

Explainer

In this packet, we will always be adding sequences that increase or decrease by the *same amount*:

4, 5, 6, 7, 8

20, 25, 30, 35

100, 98, 96, 94

These lists are not random — the difference between neighboring numbers stays the same.

Example:

$$4 + 5 + 6 + 7 + 8$$

There are 5 numbers, and the middle number is 6. The middle number is

$$\frac{4 + 8}{2} = 6.$$

We can write the sum using the middle:

$$(6 - 2) + (6 - 1) + 6 + (6 + 1) + (6 + 2)$$

Notice how the values cancel out. This is the same as adding 6 five times:

$$6 + 6 + 6 + 6 + 6 = 5 \times 6 = 30$$

What if the middle number isn't in the sum?

$$18 + 21 + 24 + 27$$

The midpoint of this sequence is

$$m = \frac{18 + 27}{2} = 22.5.$$

Write each term using the middle:

$$(22.5 - 4.5) + (22.5 - 1.5) + (22.5 + 1.5) + (22.5 + 4.5) = 4 \times 22.5 = 90$$

The Big Idea

For any list of numbers that goes up or down by the same amount between terms:

$$\boxed{\text{Sum} = n \times m}$$

where:

- n is the number of terms
- m is the middle number (the average of the first and last terms)

This works whether n is odd or even.

1. Find the sum of all multiples of 6 from 84 to 396.
2. Find the sum of all odd numbers from 101 to 299.
3. Find the sum of all multiples of 7 from 21 to 287.
4. Find the sum of all even numbers from 48 to 412.
5. Find the sum of all multiples of 3 between 100 and 2026.

11. Can 6 consecutive integers have a sum of 501? Explain.

12. Is it possible for 9 consecutive integers to have a sum of 1,000?

13. Can 8 equally spaced integers have a sum of 1,200?

14. **Triangular numbers.**

We explored triangular numbers in class and proved visually that

$$1 + 2 + 3 + \cdots + n = \frac{1}{2}n(n + 1).$$

(a) Prove this formula visually, the same way we did in class.

(b) Prove this formula using the idea $\text{Sum} = n \times m$.