Arithmetic Series – Finding the Sum

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Explainer

In this packet, we will always be adding sequences that increase or decrease by the *same amount*:

$$4, 5, 6, 7, 8 \qquad 20, 25, 30, 35 \qquad 100, 98, 96, 94$$

These lists are not random — the difference between neighboring numbers stays the same.

Example:

$$4 + 5 + 6 + 7 + 8$$

There are 5 numbers, and the middle number is 6. The middle number is

$$\frac{4+8}{2} = 6.$$

We can write the sum using the middle:

$$(6-2) + (6-1) + 6 + (6+1) + (6+2)$$

Notice how the values cancel out. This is the same as adding 6 five times:

$$6 + 6 + 6 + 6 + 6 = 5 \times 6 = 30$$

What if the middle number isn't in the sum?

$$18 + 21 + 24 + 27$$

The midpoint of this sequence is

$$m = \frac{18+27}{2} = 22.5.$$

Write each term using the middle:

$$(22.5 - 4.5) + (22.5 - 1.5) + (22.5 + 1.5) + (22.5 + 4.5) = 4 \times 22.5 = 90$$

The Big Idea
For any list of numbers that goes up or down by the same amount between terms:

$$\boxed{\text{Sum} = n \times m}$$

where:

- n is the number of terms
- m is the middle number (the average of the first and last terms)

This works whether n is odd or even.

1. Find the sum of all multiples of 6 from 84 to 396.

   Solution. $m = 240$, $n = 53$. Sum = 12,720.

2. Find the sum of all odd numbers from 101 to 299.

   Solution. $m = 200$, $n = 100$. Sum = 20,000.

3. Find the sum of all multiples of 7 from 21 to 287.

   Solution. $m = 154$, $n = 39$. Sum = 6,006.

4. Find the sum of all even numbers from 48 to 412.

   Solution. $m = 230$, $n = 183$. Sum = 42,090.

5. Find the sum of all multiples of 3 between 100 and 2026.

   Solution. $m = 1063.5$, $n = 642$. Sum = 682,767.

6. Find the sum of all multiples of 8 from 64 to 512.

   Solution. $m = 288$, $n = 57$. Sum = 16,416.

7. Find the sum of all multiples of 9 from 9 to 999.

   Solution. $m = 504$, $n = 111$. Sum = 55,944.

8. Find the sum of all multiples of 11 from 121 to 1,331.

   Solution. $m = 726$, $n = 111$. Sum = 80,586.

9. Is it possible for 4 equally spaced numbers to have a total of 500?

   Solution. $m = 500/4 = 125$. Yes, this is possible.

10. Is it possible for 7 equally spaced integers to have a total of 500?

    Solution. $m = 500/7$ is not an integer. Not possible.

11. Can 6 consecutive integers have a sum of 501? Explain.

    Solution. $m = 501/6 = 83.5$. Yes, this is possible.

12. Is it possible for 9 consecutive integers to have a sum of 1,000?

    Solution. $m = 1000/9$ is not an integer. Not possible.

13. Can 8 equally spaced integers have a sum of 1,200?

    Solution. $m = 150$. Yes, this is possible.

14. **Triangular numbers.**

    We explored triangular numbers in class and proved visually that

    $$1 + 2 + 3 + \cdots + n = \frac{1}{2}n(n+1).$$

    (a) Prove this formula visually, the same way we did in class.

    (b) Prove this formula using the idea Sum $= n \times m$.

    Solution. For $1 + 2 + \cdots + n$, the middle number is $m = \frac{1+n}{2}$ and there are n terms. So the sum is $n \times \frac{n+1}{2} = \frac{1}{2}n(n+1)$.