Two-Variable Algebra – Mixed Bag

Mr. Merrick · September 24, 2025

Explainer

Directions. Solve each problem *algebraically*. Show your steps, check your solutions, and write a clear word answer when appropriate.

Worked example (elimination). Solve for x and y:

$$3x + 4y = 26 \quad (1)$$
$$5x - 2y = 8 \quad (2)$$

To eliminate y, multiply (2) by 2 to get $10x - 4y = 16$ and add to (1):

$$(3x + 4y) + (10x - 4y) = 26 + 16 \Rightarrow 13x = 42 \Rightarrow x = \frac{42}{13}.$$

Back-substitute into (1):

$$3\left(\frac{42}{13}\right) + 4y = 26 \Rightarrow \frac{126}{13} + 4y = 26 \Rightarrow 4y = \frac{212}{13} \Rightarrow y = \frac{53}{13}.$$

(Any correct method earns full credit.)

Solving 2×2 Systems of Equations

Now solve these on your own. Confirm your solution in both equations.

(A) $3x + 2y = 17$
$5x + 7y = 43.$

Solution. From elimination, $x = 3$, $y = 4$. Check: $3(3)+2(4) = 9+8 = 17$; $5(3)+7(4) = 15+28 = 43$.

(B) $13x - 2y = 18$
$5x + 3y = 22.$

Solution. Solve to get $x = 2$, $y = 4$. Check: $13(2) - 2(4) = 26 - 8 = 18$; $5(2)+3(4) = 10+12 = 22$.

(C) $10x - 7y = 61$
$3x - 5y = -2.$

Solution. Eliminate y (e.g., multiply first by 5 and second by 7): $x = 11$, $y = 7$.

Two-variable algebra: Word problems

Solve algebraically. Define your variables, set up equations, solve, check, and give a word answer.

1. The number of students in Grade 7 and Grade 8 is the same. $\frac{2}{3}$ of the Grade 7s and all except 11 of the Grade 8s went on a museum trip. Altogether, 94 students went. How many students are in Grade 7?

   Solution. Let each grade have n students. Trip count: $\frac{2}{3}n + (n - 11) = 94 \Rightarrow \frac{5}{3}n = 105 \Rightarrow n = 63$.

2. 94 kg of rice was packed into 5 kg bags and 8 kg bags. Altogether, 14 bags were used. How many 5 kg bags were used?

   Solution. Let x be the number of 5 kg bags and y of 8 kg bags. $x + y = 14$, $5x + 8y = 94$. Solve: $x = 6$, $y = 8$.

3. A pencil and a pen together cost \$28. Three pencils and five pens cost \$104. What is the cost of a pen?

   Solution. Let pencil $= p$, pen $= q$. $p + q = 28$, $3p + 5q = 104$. Solve: $q = 10$, $p = 18$. A pen costs \$10.

4. $\frac{2}{3}$ of the apples in a crate is 14 less than $\frac{4}{5}$ of the apples in the crate. How many apples are there altogether?

   Solution. Let n be total apples. $\frac{4}{5}n - \frac{2}{3}n = 14 \Rightarrow \frac{2}{15}n = 14 \Rightarrow n = 105$.

5. A class of 48 students bought tickets for a show. Premium tickets cost \$42 each, and regular tickets cost \$26 each. The total cost was \$1,472. How many premium tickets did they buy?

   Solution. Let x premium, y regular. $x + y = 48$, $42x + 26y = 1472$. Solve: $x = 14$, $y = 34$.

6. On a contest you get 7 points for a correct answer and lose 2 points for an incorrect answer. Alisha attempted all 30 questions and scored 174 points. How many mistakes did she make?

   Solution. Let c correct, w wrong. $c + w = 30$, $7c - 2w = 174$. Solve: $w = 4$, $c = 26$.

7. A roaster bought 8.4 kg of coffee beans and sold them in 300 g bags and 500 g bags. There were 4 more 500 g bags than 300 g bags. How many bags of each size were there?

   Solution. Let x be 300 g bags, y be 500 g bags. $0.3x + 0.5y = 8.4$, $y = x + 4$. Solve: $x = 8$, $y = 12$.

8. Five cats and one dog cost \$187. Four cats and three dogs cost \$242. What is the cost of a dog?

   Solution. Let cat $= c$, dog $= d$. $5c + d = 187$, $4c + 3d = 242$. Solve: $d = 42$, $c = 29$. A dog costs \$42.

9. For \$870 you can buy 3 pairs of sneakers and 2 sets of headphones. When the sneakers go on sale for \$30 off per pair, you can buy 2 pairs of sneakers and 3 headphones for \$720. What is the cost of one set of headphones?

   Solution. Let sneakers $= s$, headphones $= h$. $3s+2h = 870$, and $(s-30)\cdot 2+3h = 720 \Rightarrow 2s+3h = 780$. Solve: $h = 120$, $s = 210$.

10. Two ice-cream cakes cost \$12 more than 5 tubs of ice cream. Three ice-cream cakes cost \$10 more than 8 tubs. How much does a single tub cost?

    Solution. Let cake $= c$, tub $= t$. $2c = 5t + 12$, $3c = 8t + 10$. Solve: $t = 16$, $c = 46$. A tub costs \$16.

11. Brad has 4 times as much money as Angelina. After she gives him \$30 million, he has 7 times as much as she does. How much money did each have at first?

    Solution. Let Angelina $= a$, Brad $= b = 4a$. After transfer: $b + 30 = 7(a - 30)$. Substitute: $4a + 30 = 7a - 210 \Rightarrow a = 80$, $b = 320$ (millions).

12. Shimon has red and black cards. If he were to give away 6 red cards, he would have twice as many black cards as red cards. If he gave away 18 black cards, he would have three times as many red cards as black. How many cards of each colour does he have?

    Solution. Let red $= r$, black $= b$. From the conditions: $b = 2(r - 6)$ and $b - 18 = \frac{1}{3}r$. Solve the system to get $r = 18$, $b = 24$.