

RATES, RATIOS, PERCENTAGES, AND PROPORTION

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Explainer

What is a Rate?

A *rate* compares two different kinds of quantities (e.g., km per hour, dollars per item, pages per minute).

$$\text{Unit Rate} = \frac{\text{Total Quantity}}{\text{Number of Units}} \quad \text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

Example (speed). A bus travels 180 km in 3 hours. Unit rate = $180 \div 3 = 60$ km/h.

Time (h)	1	2	3	4	5
Distance (km)	60	120	180	240	300

What is a Ratio?

A *ratio* compares two amounts with the same units. “ $A : B = 2 : 3$ ” means every 2 of A go with 3 of B . Equivalent ratios come from multiplying/dividing both parts by the same number.

$$A : B = x : y \iff \frac{A}{B} = \frac{x}{y}, \quad \text{Part of whole: } \frac{A}{A+B} = \frac{x}{x+y}$$

Example. Apples:Oranges = $2 : 3$.

Apples	2	4	6	8	10
Oranges	3	6	9	12	15

What is a Percentage?

“Percent” means “out of 100”. $p\% = \frac{p}{100}$.

$$\text{Increase: new} = \text{old} \times \left(1 + \frac{r}{100}\right), \quad \text{Decrease: new} = \text{old} \times \left(1 - \frac{r}{100}\right)$$

Example. A \$400 phone has a 20% discount. New price = $400 \times 0.80 = \$320$.

Direct Proportion ($y \propto x$).

Two quantities are *directly proportional* if they increase/decrease together at the same rate.

$$y \propto x \iff \frac{y_1}{y_2} = \frac{x_1}{x_2}$$

Example. 3 pens cost \$6. How much for 5 pens?

Pens (x)	1	2	3	4	5
Cost (y)	2	4	6	8	10

Inverse Proportion ($y \propto \frac{1}{x}$).

Two quantities are *inversely proportional* if one increases while the other decreases so that the product stays constant.

$$y \propto \frac{1}{x} \iff x_1 y_1 = x_2 y_2$$

Example. 4 workers take 12 days. How many days would 6 workers take (same rate)?

Workers (x)	2	4	6	8	12
Days (y)	24	12	8	6	4

True/False

1. If $A = \frac{2}{3}B$, then $B = \frac{3}{2}A$.
2. A 25% increase followed by a 25% decrease gives no net change.
3. If $A : B = 3 : 7$, then A is $\frac{3}{10}$ of the total.
4. 40% of 60 equals 60% of 40.
5. If boys:girls = 2 : 3, then girls are 60% of the class.
6. Doubling speed halves the time for the same distance.
7. The ratio 4 : 5 is equivalent to 8 : 10.
8. "Increase in the ratio 7 : 5" means a 40% increase.
9. If A is 30% of B , then $B : A = 10 : 3$.
10. If $p \propto q$, doubling p doubles q .
11. If $m \propto \frac{1}{n}$, tripling m makes n one third as large.

Word Problems (30)

Solve each problem neatly. Define variables when helpful, set up proportions/equations, and show your work. Toggle solutions on to reveal answers in green.

1. A babysitter earns \$96 for 8 hours. What is the hourly rate?
2. A printer produces 540 pages in 15 minutes. At this rate, how many pages will it produce in 35 minutes?
3. A cyclist rides 90 km in 2 hours and then 60 km in 1 hour. What is the average speed for the whole trip?
4. Divide \$360 between Sam and Tom in the ratio 2 : 7.

5. Split \$168 among A, B, and C in the ratio 2 : 5 : 7.

6. A bag contains red and blue marbles in the ratio 5 : 3. If there are 96 marbles in total, how many are red and how many are blue?

7. A map scale is 1 cm : 4 km. Two towns are 11.5 cm apart on the map. What is the real distance?

8. A jacket costs \$180 *after* a 25% discount. What was the original price?

9. A laptop's price increases from \$540 to \$621. What is the percentage increase?

10. A coat marked \$150 is discounted by 20% and then sales tax of 5% is applied to the discounted price. Find the final price.
11. A recipe uses 3 cups of flour to make 24 cookies. How many cups are needed to make 56 cookies?
12. Eight workers can finish a job in 18 days. How many days would 12 workers take, working at the same rate?
13. One tap fills a tank in 10 hours, another fills it in 15 hours. If both run together, how long to fill the tank?

14. A tap can fill a tank in 12 hours; a drain can empty it in 20 hours. If both are open, how long to fill the empty tank?
15. Two trains start toward each other from towns 300 km apart, at 70 km/h and 80 km/h. How long until they meet?
16. Car P leaves City A at 8:00 a.m. at 60 km/h. Car Q leaves City B at 8:45 a.m. at 90 km/h toward City A. If the cities are 315 km apart, at about what time do they meet?
17. Two grades have the same number of students. $\frac{2}{3}$ of Grade 7 and all except 9 of Grade 8 attended a field trip. Altogether, 101 students attended. How many students are in Grade 7?
18. A shop buys a chair for \$180 and sells it for \$225. Find the profit and percent profit.

19. An item is marked up by 25% from cost, then discounted so the final price is 10% below cost. What single discount (off the marked-up price) would achieve this?
20. A town's population grows by 6% in year 1 and 10% in year 2 to reach 58,300. What was the population two years earlier?
21. A class buys 52 tickets: premium \$45 each and regular \$28 each. The total cost is \$1,694. How many premium tickets were bought?
22. A roaster packs 8.4 kg of nuts into 300 g and 500 g bags. There are 4 more 500 g bags than 300 g bags. How many of each size?

23. In a quiz, 5 points are awarded for each correct answer and 2 points are deducted for each wrong answer. A student attempted all 40 questions and scored 144 points. How many wrong answers?
24. Mix 30% juice and 70% juice to make 2 L of 50% juice. How much of each should be used?
25. How many kilograms of 20% alloy and 35% alloy are needed to make 1.5 kg of a 28% alloy?
26. Two amounts are in the ratio 5 : 7. The larger exceeds the smaller by \$48. Find both amounts and their total.