Rates, Ratios, Percentages, and Proportion

Mr. Merrick · September 29, 2025

Explainer

What is a Rate?
A *rate* compares two different kinds of quantities (e.g., km per hour, dollars per item, pages per minute).

$$\text{Unit Rate} = \frac{\text{Total Quantity}}{\text{Number of Units}} \qquad \text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

Example (speed). A bus travels 180 km in 3 hours. Unit rate $= 180 \div 3 = 60$ km/h.

Time (h)	1	2	3	4	5
Distance (km)	60	120	180	240	300

What is a Ratio?
A *ratio* compares two amounts with the same units. "$A : B = 2 : 3$" means every 2 of A go with 3 of B. Equivalent ratios come from multiplying/dividing both parts by the same number.

$$A : B = x : y \iff \frac{A}{B} = \frac{x}{y}, \qquad \text{Part of whole: } \frac{A}{A+B} = \frac{x}{x+y}$$

Example. Apples:Oranges $= 2 : 3$.

Apples	2	4	6	8	10
Oranges	3	6	9	12	15

What is a Percentage?
"Percent" means "out of 100". $p\% = \dfrac{p}{100}$.

$$\text{Increase: new} = \text{old} \times \left(1 + \frac{r}{100}\right), \quad \text{Decrease: new} = \text{old} \times \left(1 - \frac{r}{100}\right)$$

Example. A \$400 phone has a 20% discount. New price $= 400 \times 0.80 = \$320$.

Direct Proportion ($y \propto x$).
Two quantities are *directly proportional* if they increase/decrease together at the same rate.

$$y \propto x \quad \iff \quad \frac{y_1}{y_2} = \frac{x_1}{x_2}$$

Example. 3 pens cost \$6. How much for 5 pens?

Pens (x)	1	2	3	4	5
Cost (y)	2	4	6	8	10

Inverse Proportion ($y \propto \frac{1}{x}$).
Two quantities are *inversely proportional* if one increases while the other decreases so that the product stays constant.

$$y \propto \frac{1}{x} \quad \iff \quad x_1 y_1 = x_2 y_2$$

Example. 4 workers take 12 days. How many days would 6 workers take (same rate)?

Workers (x)	2	4	6	8	12
Days (y)	24	12	8	6	4

True/False

1. If $A = \frac{2}{3}B$, then $B = \frac{3}{2}A$.

   Solution. True.

2. A 25% increase followed by a 25% decrease gives no net change.

   Solution. False; $1.25 \times 0.75 = 0.9375$ (6.25% drop).

3. If $A : B = 3 : 7$, then A is $\frac{3}{10}$ of the total.

   Solution. True.

4. 40% of 60 equals 60% of 40.

   Solution. True; both 24.

5. If boys:girls $= 2 : 3$, then girls are 60% of the class.

   Solution. True.

6. Doubling speed halves the time for the same distance.

   Solution. True; inverse proportion.

7. The ratio $4 : 5$ is equivalent to $8 : 10$.

   Solution. True.

8. "Increase in the ratio $7 : 5$" means a 40% increase.

   Solution. True; $(7 - 5)/5 = 0.4$.

9. If A is 30% of B, then $B : A = 10 : 3$.

   Solution. True.

10. If $p \propto q$, doubling p doubles q.

    Solution. True.

11. If $m \propto \frac{1}{n}$, tripling m makes n one third as large.

    Solution. True.

Word Problems (30)

Solve each problem neatly. Define variables when helpful, set up proportions/equations, and show your work. Toggle solutions on to reveal answers in green.

1. A babysitter earns \$96 for 8 hours. What is the hourly rate?

   Solution. $\$96 \div 8 = \$12$ per hour.

2. A printer produces 540 pages in 15 minutes. At this rate, how many pages will it produce in 35 minutes?

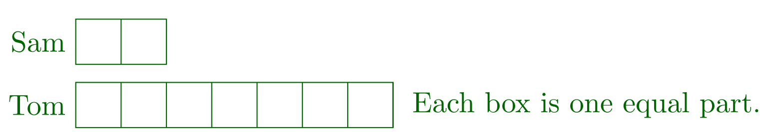
   Solution. Rate $= 540/15 = 36$ pages/min. Pages $= 36 \times 35 = 1260$.

3. A cyclist rides 90 km in 2 hours and then 60 km in 1 hour. What is the average speed for the whole trip?

   Solution. Total distance $= 150$ km; total time $= 3$ h; average speed $= 150/3 = 50$ km/h.

4. Divide \$360 between Sam and Tom in the ratio $2 : 7$.

   Solution.

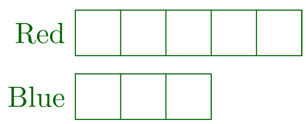
   Total parts $= 2 + 7 = 9$. One part $= 360 \div 9 = \$40$.
   Sam $= 2 \times \$40 = \$80$, Tom $= 7 \times \$40 = \$280$.

5. Split \$168 among A, B, and C in the ratio $2 : 5 : 7$.

   Solution. Think of one long bar cut into $2 + 5 + 7 = 14$ equal parts. One part $= 168 \div 14 = 12$. $A = 2 \times 12 = 24$, $B = 5 \times 12 = 60$, $C = 7 \times 12 = 84$.

6. A bag contains red and blue marbles in the ratio $5 : 3$. If there are 96 marbles in total, how many are red and how many are blue?

   Solution.

   Total parts $= 8$; one part $= 96 \div 8 = 12$.

   Red $= 5 \times 12 = 60$, Blue $= 3 \times 12 = 36$.

7. A map scale is 1 cm : 4 km. Two towns are 11.5 cm apart on the map. What is the real distance?

   Solution. $11.5 \times 4 = 46$ km.

8. A jacket costs \$180 *after* a 25% discount. What was the original price?

   Solution. Original $= 180/0.75 = \$240$.

9. A laptop's price increases from \$540 to \$621. What is the percentage increase?

   Solution. $(621 - 540)/540 = 81/540 = 0.15 = 15\%$.

10. A coat marked \$150 is discounted by 20% and then sales tax of 5% is applied to the discounted price. Find the final price.

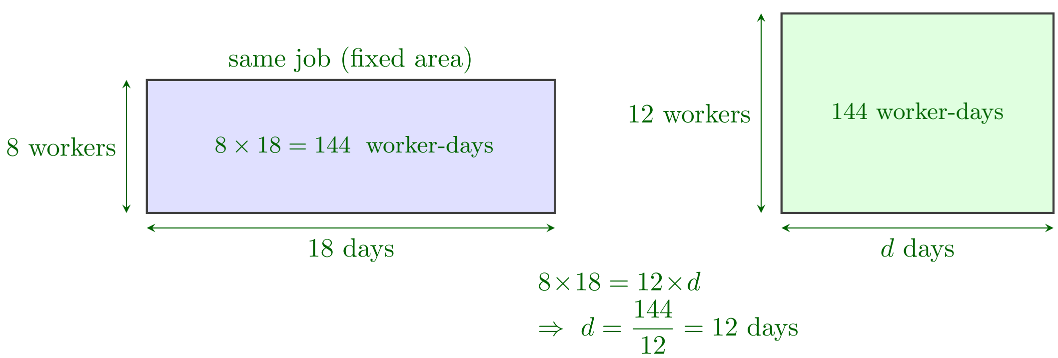
    Solution. Discounted $= 150 \times 0.80 = 120$. With tax $= 120 \times 1.05 = \$126$.

11. A recipe uses 3 cups of flour to make 24 cookies. How many cups are needed to make 56 cookies?

    Solution. Cups $= 3 \times (56/24) = 7$.

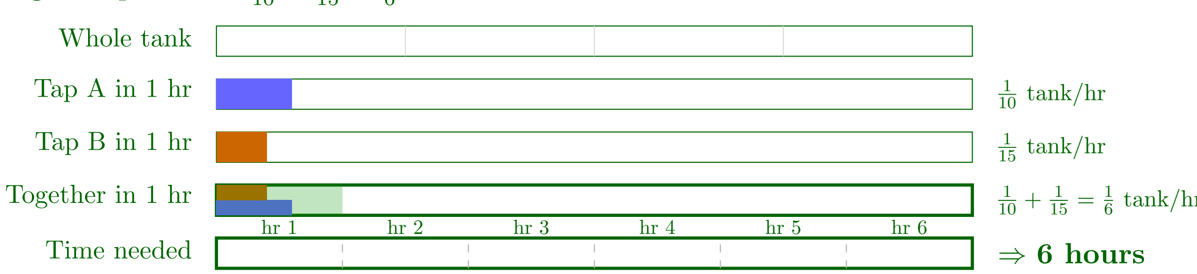
12. Eight workers can finish a job in 18 days. How many days would 12 workers take, working at the same rate?

    Solution. Inverse proportion: $8 \times 18 = 12 \times d \Rightarrow d = 12$ days.



13. One tap fills a tank in 10 hours, another fills it in 15 hours. If both run together, how long to fill the tank?

    Solution. Bar idea: Think "1 tank" as the whole bar. In one hour, Tap A fills $\frac{1}{10}$ of the bar, Tap B fills $\frac{1}{15}$.
    Together per hour: $\frac{1}{10} + \frac{1}{15} = \frac{1}{6}$ of the bar. Time for one bar = 6 hours.

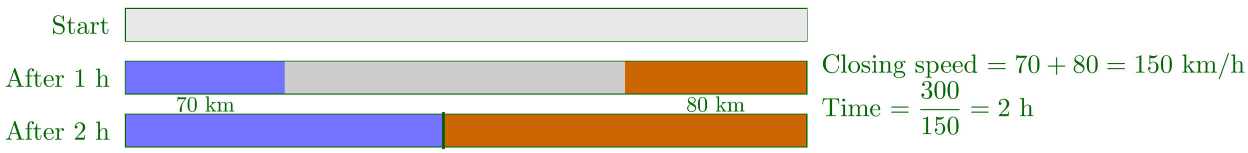


14. A tap can fill a tank in 12 hours; a drain can empty it in 20 hours. If both are open, how long to fill the empty tank?

Solution. Net rate $= 1/12 - 1/20 = (5-3)/60 = 1/30$. Time $= 30$ hours.

15. Two trains start toward each other from towns 300 km apart, at 70 km/h and 80 km/h. How long until they meet?

Solution. Bar idea: One bar = entire 300 km gap. Each hour the trains "eat" $70 + 80 = 150$ km of the gap.
Hours $= 300 \div 150 = 2$. They meet after 2 hours.



16. Car P leaves City A at 8:00 a.m. at 60 km/h. Car Q leaves City B at 8:45 a.m. at 90 km/h toward City A. If the cities are 315 km apart, at about what time do they meet?

Solution. The total gap is 315 km. P starts first and "eats" $60 \times 0.75 = 45$ km of the gap.
Remaining bar: $315 - 45 = 270$ km.
When both move, gap eaten per hour $= 60 + 90 = 150$ km/h. Time after 8:45 a.m. $= 270/150 = 1.8$ h $= 1$ h 48 min.
Meeting time $\approx$ 10:33 a.m.

17. Two grades have the same number of students. $\frac{2}{3}$ of Grade 7 and all except 9 of Grade 8 attended a field trip. Altogether, 101 students attended. How many students are in Grade 7?

Solution. Let each grade have n. $\frac{2}{3}n + (n-9) = 101 \Rightarrow \frac{5}{3}n = 110 \Rightarrow n = 66$.

18. A shop buys a chair for \$180 and sells it for \$225. Find the profit and percent profit.

Solution. Profit $= 225 - 180 = 45$. Profit% $= 45/180 = 25\%$.

19. An item is marked up by 25% from cost, then discounted so the final price is 10% below cost. What single discount (off the marked-up price) would achieve this?

Solution. Marked $= 1.25C$. Want final $= 0.90C$. Discount factor on marked price $= 0.90/1.25 = 0.72 \Rightarrow 28\%$ discount.

20. A town's population grows by 6% in year 1 and 10% in year 2 to reach 58,300. What was the population two years earlier?

Solution. Initial $= 58300/(1.06 \times 1.10) = 58300/1.166 = 50{,}000$.

21. A class buys 52 tickets: premium \$45 each and regular \$28 each. The total cost is \$1,694. How many premium tickets were bought?

Solution.

Algebra: $p + r = 52$, $45p + 28r = 1694$. Subtract $28(p + r)$: $17p = 238 \Rightarrow p = 14$.

Another Solution: If all 52 tickets were regular (\$28), the cost would be $52 \times 28 = 1456$. The actual total is \$1694, which is \$1694 − \$1456 = \$238 more than the "all-regular" baseline. Each premium costs \$17 more than a regular (\$45 − \$28 = \$17), so those \$238 extra dollars are made of \$17-chunks: $238 \div 17 = 14$ premium tickets. Then $52 - 14 = 38$ regular.

22. A roaster packs 8.4 kg of nuts into 300 g and 500 g bags. There are 4 more 500 g bags than 300 g bags. How many of each size?

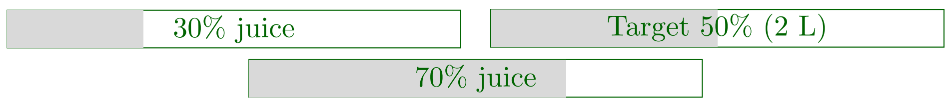
Solution. Let x be 300 g bags, $y = x + 4$ be 500 g bags. $0.3x + 0.5y = 8.4 \Rightarrow 0.3x + 0.5(x + 4) = 8.4 \Rightarrow 0.8x = 6.4$, so $x = 8$, $y = 12$.

23. In a quiz, 5 points are awarded for each correct answer and 2 points are deducted for each wrong answer. A student attempted all 40 questions and scored 144 points. How many wrong answers?

Solution. Start from the "all correct" bar: $40 \times 5 = 200$. Actual $= 144$ is 56 less.
Each wrong is a drop of 7 (since $+5$ vs -2). Wrong $= 56/7 = 8$. Algebra can also be used here.

24. Mix 30% juice and 70% juice to make 2 L of 50% juice. How much of each should be used?

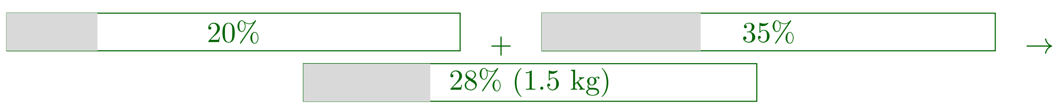
Solution.



Let x L of 30% and $2 - x$ L of 70%. $0.30x + 0.70(2 - x) = 1.00$ L $\Rightarrow x = 1.0$ L. Use 1 L of each.

25. How many kilograms of 20% alloy and 35% alloy are needed to make 1.5 kg of a 28% alloy?

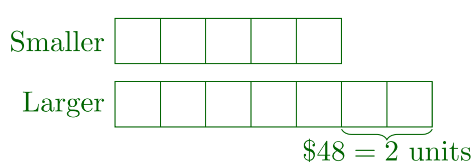
Solution.



Let x kg of 20%, $1.5 - x$ kg of 35%. $0.20x + 0.35(1.5 - x) = 0.28(1.5) \Rightarrow x = 0.7$ kg; other $= 0.8$ kg.

26. Two amounts are in the ratio $5 : 7$. The larger exceeds the smaller by \$48. Find both amounts and their total.

Solution.



"Extra" $= 7 - 5 = 2$ units $=$ \$48 $\Rightarrow$ one unit $=$ \$24.
Smaller $= 5 \times 24 =$ \$120, Larger $= 7 \times 24 =$ \$168, Total $=$ \$288.