Word Problems with Averages

Mr. Merrick · September 23, 2025

Explainer

Reminder: The Arithmetic Mean (Average).
The *average* (or arithmetic mean) of several numbers is found by dividing the total of the numbers by how many numbers there are:

$$\text{Average} = \frac{\text{Sum of all values}}{\text{Number of values}}.$$

Examples:

- The average of $4, 7, 9$ is $\frac{4+7+9}{3} = \frac{20}{3} \approx 6.7$.
- If the average of 5 numbers is 12, then their total sum is $5 \times 12 = 60$.

In word problems: Often you'll first find a *total* using average × number of items, or set up an equation equating two totals. This is the key move throughout this set.

1. A school cafeteria prepared 120 sandwiches. The average cost of the vegetarian sandwiches was \$2.40, while the average cost of the non-vegetarian sandwiches was \$3.10. If the overall average cost of all sandwiches was \$2.75, how many vegetarian sandwiches were there?

   Solution. Let v = vegetarian. Then $2.40v + 3.10(120 - v) = 2.75 \cdot 120$. Solving gives $v = 60$.

2. A class of 25 students has an average height of 150 cm. After 5 new students joined, the class average increased to 152 cm. What was the average height of the new students?

   Solution. Old total $= 25 \cdot 150 = 3750$. New total $= 30 \cdot 152 = 4560$. Extra $= 810$, so average $= 810/5 = 162$ cm.

3. On the first five tests, a student's average is 72. What score is needed on the sixth test to raise the overall average to 75?

   Solution. Desired total $= 6 \cdot 75 = 450$. Current total $= 5 \cdot 72 = 360$. Needed score $= 450 - 360 = \boxed{90}$.

4. The average of 12 numbers is 18. When one number is removed, the average of the remaining 11 numbers is 16. What number was removed?

   Solution. Original sum $= 12 \cdot 18 = 216$. New sum $= 11 \cdot 16 = 176$. Removed number $= 216 - 176 = \boxed{40}$.

5. The average of 6 test scores is 75. When the highest score is removed, the average of the remaining 5 scores drops to 72. When the lowest score is removed instead, the average of the remaining 5 scores rises to 78. What were the highest and lowest scores?

   Solution. Total sum $T = 6 \cdot 75 = 450$. Let H = highest, L = lowest.

   Removing highest: $\frac{450-H}{5} = 72 \Rightarrow H = 90$. Removing lowest: $\frac{450-L}{5} = 78 \Rightarrow L = 60$.

   So the highest score is 90 and the lowest is 60.

6. The average age of a group of 8 friends is 21 years. When two more friends join, the average becomes 22 years. What is the average age of the two new friends?

   Solution. Old total = 168, new total = 220, added = 52. Average of the two = $52/2 = 26$ years.

7. A chemist mixes 4 L of a 25% alcohol solution with some amount of a 40% alcohol solution to obtain a 32.5% alcohol solution. How many liters of the 40% solution should be added?

   Solution. Let y be liters of 40%. $0.25 \cdot 4 + 0.40y = 0.325(4 + y) \Rightarrow 1 + 0.40y = 1.3 + 0.325y \Rightarrow 0.075y = 0.3 \Rightarrow y = \boxed{4}$ L.

8. The average score of three students, Alan, Ben, and Chris, is 78. The average of Alan and Ben is 82. What is Chris's score?

   Solution. Total $= 3 \cdot 78 = 234$, Alan+Ben $= 2 \cdot 82 = 164$. Chris $= 234 - 164 = \boxed{70}$.

9. A company has two departments. The average monthly salary in Department A is \$4800, while in Department B it is \$5200. If the overall average is \$5000 and there are 30 employees in Department A, how many employees are in Department B?

   Solution. $4800 \cdot 30 + 5200x = 5000(30 + x) \Rightarrow x = \boxed{30}$.