Set Puzzles
September 23, 2025

1. A set of three *distinct* positive integers has mean 6, median 7, and range 7. What is the smallest number in the set?

   Solution. Let the sorted set be $a < b < c$ with $b = 7$, $a + c = 18 - 7 = 11$, and $c - a = 7$. Solving gives $c = 9$, $a = 2$. Smallest $= 2$.

2. In a set of four numbers, the mean, median, and range are all 40. What is the greatest number in the set?

   Solution. If $x_1 \le x_2 \le x_3 \le x_4$, then $x_2 + x_3 = 80$, $x_4 - x_1 = 40$, and $x_1 + x_2 + x_3 + x_4 = 160$. Hence $x_1 + x_4 = 80$ and together with the range, $2x_4 = 120 \Rightarrow x_4 = 60$.

3. Ten numbers are listed from least to greatest so that the average of the first n numbers is always $2n$ for $1 \le n \le 10$. What is the 10^{th} number?

   Solution. If $S_n = 2n^2$, then $a_n = S_n - S_{n-1} = 2n^2 - 2(n-1)^2 = 4n - 2$. Thus $a_{10} = 38$.

4. In the sequence below, each term beginning with b equals the average of the two preceding terms. Find a.

   $$80,\ a,\ b,\ c,\ 50$$

   Solution. $b = \frac{80+a}{2}$, $c = \frac{a+b}{2}$, and $50 = \frac{b+c}{2}$. This yields $a = \dfrac{8 \cdot 50 - 3 \cdot 80}{5} = 32$.

5. Five numbers will be selected from the set $\{1, 4, 6, 8, 9, 11, 15\}$ so that the mean of the five numbers is 9. *Which two numbers must be excluded?*

   Solution. Target sum $= 45$. Total sum is 54, so the two excluded must sum to 9. The only pair is $\{1, 8\}$.

6. The range of the set $\{12, 20, 18, 85, 37, 32, n\}$ is 80. What is the sum of all possible values of n?

   Solution. Either n is new max with min 12: $n - 12 = 80 \Rightarrow n = 92$, or n is new min with max 85: $85 - n = 80 \Rightarrow n = 5$. Sum $= 97$.

7. A set of five integers has smallest value 7, median 19, and range 26. What is the largest value in the set?

   Solution. Largest = smallest + range $= 7 + 26 = 33$.

8. In a set of five integers, the *unique* mode is 30, the median is 50, and the mean is 62. What is the largest possible number in the set?

   Solution. Arrange as $30, 30, 50, d, e$ to keep 30 the unique mode and maximize e by minimizing $d > 50$. Take $d = 51$. Sum is 310, so $e = 310 - (30 + 30 + 50 + 51) = 149$.

9. In a set of six integers, the mean, median, range, and *unique* mode are all 12. What is the largest possible element?

   Solution. Let $a \le b \le c \le d \le e \le f$, with $c + d = 24$, $f - a = 12$, and sum $= 72$. A maximizing configuration is $(8, 8, 12, 12, 12, 20)$, which has unique mode 12, range 12, and total 72. Thus $f = 20$ is maximal.

10. The set $\{8, 14, 20, 26, n\}$ has the same mean and median. What is the *sum of all possible* values of n?

Solution. Mean $= (68 + n)/5$. Considering where n falls in the order: if $n \leq 8$, median $= 14 \Rightarrow n = 2$; if $14 < n < 20$, median $= n \Rightarrow (68 + n)/5 = n \Rightarrow n = 17$; if $n \geq 26$, median $= 20 \Rightarrow n = 32$. Sum $= 2 + 17 + 32 = 51$.

11. The set $\{50, 70, 85, 95, n\}$ has the same mean and median. What is the *sum of all possible* values of n?

Solution. Mean $= (300 + n)/5$. Checking positions of n gives solutions $n \in \{50, 75, 125\}$. Sum $= 250$.

12. The set $\{64, 80, 41, 27, 99, 91, n\}$ has the same median and *unique* mode. What is the *sum of all possible* values of n?

Solution. Sorting with various n shows the only possibilities making the (unique) mode equal to the median are $n = 64$ and $n = 80$. Sum $= 144$.

13. The set $\{72, 82, 92, 172, n\}$ has its mean equal to its range. What is the integer n?

Solution. Range $=$ max $-$ min, mean $= \dfrac{418 + n}{5}$. Checking cases shows the only value satisfying mean $=$ range is $n = 82$.

14. The set $\{15, 19, n, 31\}$ has the same mean and median. What is the *sum of all possible* values of n?

Solution. Let the sorted middle two average equal the mean $\left(\frac{65+n}{4}\right)$. Casework on n yields $n \in \{3, 27, 35\}$, whose sum is 65.