

Extra Practice: Prime Factors, Applications, Rational/Irrational, Number Systems & Radicals

Math 10 · Mr. Merrick

Prime Factors

- State *all* positive divisors of the following.
 - 84 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84
 - 75 1, 3, 5, 15, 25, 75
 - 96 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96
 - 105 1, 3, 5, 7, 15, 21, 35, 105
- In each case, determine the *number* of factors of the given whole number.
 - 96 $96 = 2^5 \cdot 3 \Rightarrow (5 + 1)(1 + 1) = 12$
 - 131 *Prime* $\Rightarrow 2$ factors
 - 225 $225 = 3^2 \cdot 5^2 \Rightarrow (2 + 1)(2 + 1) = 9$
 - 256 $256 = 2^8 \Rightarrow 9$ factors
 - 374 $374 = 2 \cdot 11 \cdot 17 \Rightarrow 2 \cdot 2 \cdot 2 = 8$
- From the list in Question 2, state which numbers are prime and which are composite. *Prime: 131. Composite: 96, 225, 256, 374.*
- Classify each whole number as prime or composite.
 - 47
 - 91
 - 101
 - 143
 - 221
 - 257*(a) prime; (b) $7 \cdot 13$ composite; (c) prime; (d) $11 \cdot 13$ composite; (e) $13 \cdot 17$ composite; (f) prime*
- Twin primes are consecutive odd primes (e.g. 5, 7). List seven other twin-prime pairs < 120. (11, 13), (17, 19), (29, 31), (41, 43), (59, 61), (71, 73), (101, 103)
- State the factors of 48.
1, 2, 3, 4, 6, 8, 12, 16, 24, 48
 - State the *prime* factors of 48. 2, 3
 - Express 72 as a product of prime factors.
 $72 = 2^3 \cdot 3^2$
- State the *prime factors* of:
 - 18 2, 3
 - 40 2, 5
 - 63 3, 7
 - 90 2, 3, 5
- Explain why the numbers 0 and 1 have no prime factors. *0 is divisible by every integer (no unique prime factorization). 1 has only one factor (itself) and no prime divisors.*
- Use a *division table* to determine the prime factorization of:
 - 252 $2^2 \cdot 3^2 \cdot 7$
 - 378 $2 \cdot 3^3 \cdot 7$
 - 2025 $3^4 \cdot 5^2$
 - 2926 $2 \cdot 7 \cdot 11 \cdot 19$
- Use a *factor tree* to determine the prime factorization of:
 - 784 $2^4 \cdot 7^2$
 - 960 $2^6 \cdot 3 \cdot 5$
 - 4725 $3^3 \cdot 5^2 \cdot 7$
 - 8400 $2^4 \cdot 3 \cdot 5^2 \cdot 7$
- In each case, write the number as a product of prime factors.
 - 3315 $3 \cdot 5 \cdot 13 \cdot 17$
 - 8085 $3 \cdot 5 \cdot 7^2 \cdot 11$
 - 9990 $2 \cdot 3^3 \cdot 5 \cdot 37$
 - 7980 $2^2 \cdot 3 \cdot 5 \cdot 7 \cdot 19$
- Which of the following numbers is *not* a prime factor of 2079?
A. 3 B. 7 C. 11 D. 13 $2079 = 3^3 \cdot 7 \cdot 11$; *not a factor: 13.*
- How many numbers in the list 2, 3, 9, 13 are *not* prime factors of 2592? $2592 = 2^5 \cdot 3^4$; *not prime factors: 9, 13 $\Rightarrow 2$*
- The sum of all *distinct* prime factors of 462 462 is _____. $462 \cdot 462 = 2 \cdot 3 \cdot 7 \cdot 11 \cdot 13 \Rightarrow 36$
- There is only one set of *prime triplets* (three consecutive odd primes). If the triplets are a, b, c , find abc . $(3, 5, 7) \Rightarrow 105$
- The number 375 can be expressed as $p \times q^r$ in primes. Find $p+q+r$. $375 = 3 \cdot 5^3 \Rightarrow 3+5+3 = 11$

Applications of Prime Factors

- State the greatest common factor (GCF) of:
 - 18 and 27 **9**
 - 32 and 56 **8**
 - 36, 48, 90 **6**
- Use prime factorization to determine the GCF of:
 - 180 and 420 $180 = 2^2 \cdot 3^2 \cdot 5$, $420 = 2^2 \cdot 3 \cdot 5 \cdot 7 \Rightarrow GCF = 60$
 - 294 and 385 $294 = 2 \cdot 3 \cdot 7^2$, $385 = 5 \cdot 7 \cdot 11 \Rightarrow GCF = 7$
 - 252 and 756 $252 = 2^2 \cdot 3^2 \cdot 7$, $756 = 2^2 \cdot 3^3 \cdot 7 \Rightarrow GCF = 252$
- Use prime factorization to determine the GCF of each pair.
 - 528 and 780 $528 = 2^4 \cdot 3 \cdot 11$, $780 = 2^2 \cdot 3 \cdot 5 \cdot 13 \Rightarrow 12$
 - 616 and 840 $616 = 2^3 \cdot 7 \cdot 11$, $840 = 2^3 \cdot 3 \cdot 5 \cdot 7 \Rightarrow 56$
 - 1870 and 2210 $1870 = 2 \cdot 5 \cdot 11 \cdot 17$, $2210 = 2 \cdot 5 \cdot 13 \cdot 17 \Rightarrow 170$
 - 714 and 1050 $714 = 2 \cdot 3 \cdot 7 \cdot 17$, $1050 = 2 \cdot 3 \cdot 5^2 \cdot 7 \Rightarrow 42$
 - 128 and 320 2^7 and $2^6 \cdot 5 \Rightarrow 64$
 - 735 and 980 $735 = 3 \cdot 5 \cdot 7^2$, $980 = 2^2 \cdot 5 \cdot 7^2 \Rightarrow 245$
- Determine the GCF of:
 - 84, 420, 1008 **84**
 - 128, 984, 1496, 3080 **8**
- State the lowest common multiple (LCM) of:
 - 8 and 12 **24**
 - 7 and 9 **63**
 - 12 and 20 **60**
 - 15 and 18 **90**
- Use prime factorization to determine the LCM of:
 - 18 and 24 $2^3 \cdot 3^2 = 72$
 - 45 and 84 $2^2 \cdot 3^2 \cdot 5 \cdot 7 = 1260$
 - 96 and 144 $2^5 \cdot 3^2 = 288$
 - 55 and 143 $5 \cdot 11 \cdot 13 = 715$
 - 72 and 252 $2^3 \cdot 3^2 \cdot 7 = 504$
- Determine the LCM of:
 - 8, 12, 18 **72**
 - 6, 14, 35 **210**
 - 9, 10, 25 **450**
 - 12, 30, 105 **420**
- In each case, decide whether the number is a perfect square (give the root if so).
 - 9801 **99^2**
 - 7776 *not a square ($2^5 \cdot 3^5$)*
 - 4900 **70^2**
 - 1089 **33^2**
- Consider 103 823.
 - Evaluate $\sqrt[3]{103\,823}$. **47**
 - Explain why 103 823 is a perfect cube. **47^3 ; prime exponents are all multiples of 3.**
- Use prime factorization to test for perfect cubes (give the cube root if so).
 - 2744 **14^3**
 - 110 592 **48^3**
 - 35 937 **33^3**
 - 421 875 **75^3**
- Explain how to tell if a number is *both* a perfect square and cube. **All prime exponents multiples of 6 (a perfect 6th power).**
- The greatest common factor of 425 and 595 is **A. 5 B. 7 C. 17 D. 85** $425 = 5^2 \cdot 17$, $595 = 5 \cdot 7 \cdot 17 \Rightarrow 85$.
- Two whole numbers x, y have $\gcd(x, y) = 14$. Which statement must be false? **A. x, y both even B. xy divisible by 98 C. x, y both multiples of 7 D. Neither x nor y can be prime (B) is false; $\gcd = 14$ does not force 7^2 in xy .**
- The LCM of 36, 231, 275 is _____. $36 = 2^2 \cdot 3^2$, $231 = 3 \cdot 7 \cdot 11$, $275 = 5^2 \cdot 11 \Rightarrow LCM = 2^2 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11 = 69,300$
- An encyclopedia has 840 pages. Page 12 and every 12th page is green; page 21 and every 21st is orange. How many pages are both? **Multiples of $\text{lcm}(12, 21) = 84$ up to 840: 10 pages.**

Rational and Irrational Numbers

- For each, state repeating/non-repeating and terminating/non-terminating.
 - $\frac{7}{20}$ 0.35; *terminating*
 - 0.742742742... *repeating, non-terminating*
 - $\frac{19}{22}$ *repeating, non-terminating*
 - $\sqrt{\frac{196}{400}}$ $\frac{7}{10}$; *terminating*
 - $-\sqrt{31}$ *non-repeating, non-terminating (irrational)*
 - $\sqrt{0.36}$ 0.6; *terminating*
 - $-4\frac{5}{11}$ $-4.\overline{45}$; *repeating*
 - π *non-repeating, non-terminating (irrational)*
- True/False.
 - Every terminating decimal is rational. *T*
 - A repeating decimal cannot be written as a fraction. *F*
 - Only terminating decimals are rational. *F*
 - Every rational decimal is either terminating or repeating. *T*
 - A decimal cannot be both repeating and non-repeating. *T*
 - π is irrational. *T*
- Rational or irrational? Briefly justify.
 - $-\frac{17}{8}$ *rational; ratio of integers*
 - 0.605 *rational; terminating*
 - $\sqrt{196}$ *rational; 14*
 - 0.305305305... *rational; repeating*
- Order on a number line:
 $\sqrt{14}$, $\sqrt{\pi}$, $\sqrt{0.2}$, $\sqrt{98}$, $2\sqrt{11}$, $3\sqrt{5}$.
Approximations: $\sqrt{0.2} \approx 0.447$,
 $\sqrt{\pi} \approx 1.772$,
 $\sqrt{14} \approx 3.742$,
 $2\sqrt{11} \approx 6.633$,
 $3\sqrt{5} \approx 6.708$,
 $\sqrt{98} \approx 9.899$.
So $\sqrt{0.2} < \sqrt{\pi} < \sqrt{14} < 2\sqrt{11} < 3\sqrt{5} < \sqrt{98}$.
- Identify as rational or irrational; if rational, simplest fraction.
 - 0.92 $\frac{23}{25}$
 - $\sqrt{\frac{9}{121}}$ $\frac{3}{11}$
 - $\sqrt{0.0121}$ $0.11 = \frac{11}{100}$
 - $-\sqrt{97}$ *irrational*
 - $-0.\overline{8} - \frac{8}{9}$
 - $-\sqrt{\frac{49}{81}}$ $-\frac{7}{9}$
 - $4.612612\ldots$ $\frac{512}{111}$
 - $\sqrt{\frac{361}{529}}$ $\frac{19}{23}$
 - $5.\overline{0}$ 5
- Convert to improper fraction (simplest form).
 - $0.\overline{7}$ $\frac{7}{9}$
 - $0.1\overline{6}$ $\frac{1}{6}$
 - $1.2\overline{3}$ $\frac{37}{30}$
 - $0.\overline{204}$ $\frac{204}{999} = \frac{68}{333}$
 - $-2.45\overline{45}$ $-\frac{245}{99}$
- Convert the repeating decimal to a fraction (algebraic method).
 - $0.\overline{3}$ $\frac{1}{3}$
 - $0.7\overline{2}$ $\frac{13}{18}$
 - $0.009\overline{81}$ $\frac{27}{2750}$
- Convert each terminating decimal to an improper fraction (lowest terms).
 - 3.007 $\frac{3007}{1000}$
 - $-2.125 - \frac{17}{8}$
 - 4.0625 $\frac{65}{16}$
- The decimal for $\frac{7}{12}$ is
A. terminating & repeating
B. terminating & non-repeating
C. non-terminating & repeating
D. non-terminating & non-repeating
Correct: C.
- Which is irrational?
A. $\sqrt{256}$ **B.** $\sqrt{0.09}$ **C.** $\frac{25}{6}$ **D.** $\sqrt{50}$
Correct: D.
- $9.\overline{9}$ is equal to
A. $\frac{99}{10}$ **B.** $\frac{999}{100}$ **C.** 10 **D.** 9 *Correct: C (since $9.\overline{9} = 10$).*
- Write $0.\overline{27} = \frac{a}{b}$ in lowest terms and compute $b - a$. $\frac{3}{11} \Rightarrow 8$

Number Systems

- Place each into the appropriate nested sets ($N \subset W \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$ and $\mathbb{R} \setminus \mathbb{Q}$):
 $-3, \sqrt{81}, \frac{29}{11}, \sqrt{2}, 0, \pi$.
 $-3 : \mathbb{Z}; \sqrt{81} = 9 : N; \frac{29}{11} : \mathbb{Q}; \sqrt{2} : \mathbb{R} \setminus \mathbb{Q}; 0 : W;$
 $\pi : \mathbb{R} \setminus \mathbb{Q}$.
- List all sets (largest \rightarrow smallest) each belongs to.
 - $-8 \mathbb{R}, \mathbb{Q}, \mathbb{Z}$
 - $\sqrt{64} \mathbb{R}, \mathbb{Q}, \mathbb{Z}, W, N$
 - $3.2727 \dots \mathbb{R}, \mathbb{Q}$
 - $-\frac{12}{7} \mathbb{R}, \mathbb{Q}$
 - $0 \mathbb{R}, \mathbb{Q}, \mathbb{Z}, W$
 - $\sqrt{11} \mathbb{R} \setminus \mathbb{Q}$
 - non-repeating $-2.1345218 \dots \mathbb{R} \setminus \mathbb{Q}$
 - $\pi \mathbb{R} \setminus \mathbb{Q}$
- Why does -7 belong to more sets than $-\frac{7}{2}$? -7 is an integer (hence rational, real); $-\frac{7}{2}$ is not an integer.
- Indicate membership in $N, W, \mathbb{Z}, \mathbb{Q}, \overline{\mathbb{Q}}$ (irrationals), and \mathbb{R} for each.
 - $\frac{1}{5} \mathbb{Q}, \mathbb{R}$
 - $123987 N, W, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$
 - $-4 \mathbb{Z}, \mathbb{Q}, \mathbb{R}$
 - $7.534 \mathbb{Q}, \mathbb{R}$
 - $9.5 \mathbb{Q}, \mathbb{R}$
 - $\sqrt{75} \overline{\mathbb{Q}}, \mathbb{R}$
 - $-\pi \overline{\mathbb{Q}}, \mathbb{R}$
 - $-\frac{355}{113} \mathbb{Q}, \mathbb{R}$
 - $-\sqrt{49} \mathbb{Z}, \mathbb{Q}, \mathbb{R}$
 - $0.000005 \mathbb{Q}, \mathbb{R}$
 - non-repeating $2.232425 \dots \overline{\mathbb{Q}}, \mathbb{R}$
- Find a number that satisfies each condition.
 - Integer but not whole. -3
 - Rational but not integer. $\frac{7}{5}$
 - Real but not rational. $\sqrt{5}$
 - Whole but not natural. 0
- Fill with *always/sometimes/never*.
 - A whole number is _____ a natural number. *Sometimes (0 excluded)*
 - The quotient of two integers is _____ an integer. *Sometimes*
 - A whole number is _____ a rational number. *Always*
 - The difference between two integers is _____ an integer. *Always*
 - The square root of a number is _____ irrational. *Sometimes*
 - A negative number is _____ in W . *Never*
 - A number in N is _____ in \mathbb{R} . *Always*
- True/False.
 - All natural numbers are integers. *T*
 - Real numbers consist of rationals and irrationals. *T*
 - Integers are nested within rationals. *T*
 - All integers are rational. *T*
 - All irrationals are real. *T*
 - \mathbb{R} is contained in N . *F*
 - \mathbb{Q} is contained in W . *F*
 - Exactly one element of W is not in N . *T (0)*
- More about roots (True/False).
 - Every positive number has two square roots but one cube root. *T (real roots)*
 - Every negative number has one real cube root but no real square roots. *T*
- Short explanations (estimation).
 - $\sqrt{8} + \sqrt{17} \neq \sqrt{25}$ $2.83 + 4.12 \approx 6.95 \neq 5$
 - $\sqrt{2} + \sqrt{3} + \sqrt{4} \neq \sqrt{9}$ $1.41 + 1.73 + 2 = 5.14 \neq 3$
- Determine true or false.
 - $\sqrt{9} + \sqrt{4} = \sqrt{9+4}$. *F*
 - $\sqrt{9} - \sqrt{4} = \sqrt{9-4}$. *F*
 - $\sqrt{9} \cdot \sqrt{4} = \sqrt{36}$. *T*
 - $\sqrt{9} + \sqrt{4} = \sqrt{9} + 4$. *F*

11. For each, (i) estimate mentally; (ii) use a calculator (nearest tenth) and judge the estimate.
- $\sqrt{21}$ (i) 4-5; (ii) ≈ 4.6
 - $\sqrt{27.4}$ (i) 5-6; (ii) ≈ 5.2
 - $4\sqrt{48} - 3\sqrt{63} \approx 3.9$
 - $\frac{3}{4}\sqrt{14.2} + \frac{1}{2}\sqrt{5} \approx 4.0$
 - $\sqrt{123} \approx 11.1$
 - $\sqrt{\sqrt{90}} \approx 3.08$
 - $\sqrt{10} + \sqrt{24.5} \approx 8.11$
 - $\sqrt{\sqrt{2601}} \approx 7.14$
12. Estimate to one significant digit.
- $\sqrt{507.1} \approx 20$
 - $\sqrt{7991} \approx 90$
 - $\sqrt{10\,389} \approx 100$
 - $\sqrt{823\,775} \approx 900$
 - $\sqrt{0.501} \approx 0.7$
 - $\sqrt{0.0501} \approx 0.2$
 - $\sqrt{0.0876} \approx 0.3$
 - $\sqrt{0.000\,397\,2} \approx 0.02$
13. (i) estimate; (ii) calculator (nearest tenth).
- $\sqrt[3]{25} \approx 2.9$
 - $\sqrt[3]{2} \approx 1.3$
 - $\sqrt[3]{202} \approx 5.9$
 - $\sqrt[3]{999.9} \approx 10.0$
 - $2\sqrt[3]{58.7} - 3\sqrt[3]{7.62} \approx 1.9$
 - $\frac{2}{3}\sqrt{40} - \frac{1}{2}\sqrt{60} \approx 0.3$
 - $\sqrt[3]{3\sqrt{10}} \approx 2.1$
14. Order on the number line:
 $\sqrt{50}$, $\sqrt[3]{50}$, $5\sqrt{10}$, $\sqrt[3]{10^3}$, $10\sqrt{5}$, $10\sqrt[3]{5}$.
 $\sqrt[3]{50} \approx 3.68 < \sqrt{50} \approx 7.07 < \sqrt[3]{1000} = 10 < 5\sqrt{10} \approx 15.81 < 10\sqrt[3]{5} \approx 17.10 < 10\sqrt{5} \approx 22.36$.
15. Which nesting statement is *false*?
- Integers \subset rationals
 - Naturals \subset wholes
 - Irrationals \subset reals
 - Reals \subset naturals *Correct: D.*
16. How many of $-\sqrt{6}$, $\sqrt[3]{-6}$, $-\sqrt[3]{6}$, $\sqrt{-6}$ are not real? $\sqrt{-6}$ only $\Rightarrow 1$
17. How many of $\sqrt{49}$, $\sqrt{49/100}$, $\sqrt{0.49}$, $\sqrt{\frac{4}{9}}$ can be written as $\frac{a}{b}$ with $a, b \in \mathbb{N}$? *All 4.*
18. To the nearest hundredth, evaluate $5\sqrt[3]{7}$. ≈ 9.57
19. Evaluate the absolute values.
- $|-4|$ 4
 - $|13|$ 13
 - $|3 - 9|$ 6
 - $||3| - |9||$ 6
 - $|\sqrt[3]{27}|$ 3
 - $|\sqrt[3]{-27}|$ 3
20. Decide whether the statement is true or false.
- $|x| = x$ if $x > 0$. *T*
 - $|x| = -x$ if $x < 0$. *T*
21. Sketch solution sets on a number line.
- $|x| < 5$ $(-5, 5)$
 - $|a| \geq 3$ $(-\infty, -3] \cup [3, \infty)$

Radicals

- Mentally evaluate where possible (real numbers).
 - $\sqrt{81}$ 9
 - $\sqrt[4]{81}$ 3
 - $5\sqrt[3]{27}$ 15
 - $\sqrt[5]{100\,000}$ 10
 - $\sqrt{\frac{16}{25}}$ $\frac{4}{5}$
 - $\sqrt[4]{\frac{1}{16}}$ $\frac{1}{2}$
 - $4\sqrt[4]{\frac{1}{16}}$ 2
 - $-\sqrt{1}$ -1
 - $\sqrt{-1}$ *not real*
 - $\sqrt[5]{-1}$ -1
 - $7\sqrt[3]{-125}$ -35
 - $-\sqrt[4]{\frac{1}{16}}$ $-\frac{1}{2}$
 - $3\sqrt{144}$ 36
 - $\frac{5}{2}\sqrt[5]{32}$ 5
 - $-\sqrt[11]{-1}$ 1 (*since $\sqrt[11]{-1} = -1$, so $-(-1) = 1$*)
 - $\sqrt[3]{\frac{8}{27}}$ $\frac{2}{3}$
- True/False.
 - The square roots of 25 are ± 5 . *T*
 - $\sqrt{25} = \pm 5$. *F (principal root = +5)*
 - If $x^2 = 25$ and $x \in \mathbb{R}$, then $x = \pm 5$. *T*
- Use a calculator to evaluate (state sign first, then value as needed).
 - $\sqrt[4]{4096}$ 8
 - $\sqrt[5]{-243}$ -3
 - $-\sqrt[4]{2401}$ -7
 - $-\sqrt[3]{729}$ -9
 - $\sqrt[3]{-729}$ -9
 - $-8\sqrt[4]{\frac{1}{256}}$ -1
 - $\sqrt[6]{0.015625}$ 0.5
 - $\sqrt[4]{-6561}$ *not real*
 - $\frac{3}{2}\sqrt[4]{\frac{16}{81}}$ 1
- Evaluate to the nearest hundredth.
 - $\sqrt[4]{10} \approx 1.78$
 - $\sqrt[8]{29} \approx 1.54$
 - $\frac{3}{2}\sqrt[3]{-527} \approx -12.12$
- Evaluate to the nearest tenth.
 - $\sqrt[5]{-25} \approx -1.9$
 - $-5\sqrt[4]{169} \approx -18.0$
 - $\frac{1}{2}\sqrt[3]{-81} \approx -2.2$
- Identify the *index* and the *radicand* in each radical.
 - $\sqrt[3]{42}$
index: _____
radicand: _____
index 3, radicand 42
 - $\sqrt[4]{36}$
index: _____
radicand: _____
index 4, radicand 36
 - $5\sqrt{17}$
index: _____
radicand: _____
index 2, radicand 17; 5 is a coefficient
- Explain the meaning of the index 4 in the radical $\sqrt[4]{36}$. *It means the fourth root: the number which, raised to the 4th power, equals 36.*
- Determine whether each statement is **true** or **false**.
 - $\sqrt{30} = \sqrt{5}\sqrt{6}$ *True*
 - $\sqrt{6-4} = \sqrt{6} - \sqrt{4}$ *False*
 - $\sqrt{3} = \frac{\sqrt{45}}{\sqrt{15}}$ *True*
 - $\frac{\sqrt{20}}{\sqrt{10}} = \sqrt{10}$ *False*
 - $\sqrt{2} + \sqrt{2} = \sqrt{4}$ *False*
 - $\sqrt{2} \times \sqrt{2} = \sqrt{4}$ *True*
 - $\sqrt{\frac{1}{2} \cdot 30} = \sqrt{15}$ *True*
 - $\frac{1}{2}\sqrt{30} = \sqrt{15}$ *False*

9. Write as a *single* radical in the form \sqrt{x} (simplify x).
- $\sqrt{5} \sqrt{7} \sqrt{35}$
 - $\sqrt{14} \sqrt{2} \sqrt{28}$
 - $\sqrt{3} \cdot \sqrt{8} \sqrt{24}$
 - $\sqrt{6} \cdot \sqrt{11} \sqrt{66}$
 - $\frac{\sqrt{20}}{\sqrt{10}} \sqrt{2}$
 - $\frac{\sqrt{25}}{\sqrt{5}} \sqrt{5}$
 - $\frac{\sqrt{10} \sqrt{6}}{\sqrt{2}} \sqrt{30}$
 - $\frac{\sqrt{81}}{\sqrt{9}} \sqrt{9}$
10. Express each as a product of radicals (split into two square roots).
- $\sqrt{35} \sqrt{5} \sqrt{7}$
 - $\sqrt{33} \sqrt{3} \sqrt{11}$
 - $\sqrt{65} \sqrt{5} \sqrt{13}$
 - $\sqrt{49} \sqrt{7} \sqrt{7}$
11. Consider the statements:
- The cube root of -27 (over the reals) is ± 3 .
 - The fourth roots of 81 (over the reals) are ± 3 .
 - $-\sqrt[3]{1000} = \sqrt[3]{-1000}$.
 - $-\sqrt[4]{16} = \sqrt[4]{-16}$.
- Which are true?
- II and III only
 - I, II, and III only
 - I, II, III, and IV
 - Some other combination *Correct: A. (I false, II true, III true, IV false).*
12. In the radical $\sqrt[4]{18}$, the index and radicand are
- index 2, radicand $\sqrt{18}$
 - index 1, radicand 1
 - index 18, radicand 1
 - index 4, radicand 18 *Correct: D.*
13. To the nearest hundredth, evaluate $\sqrt{\frac{7}{8}} + 2\sqrt[4]{\frac{7}{8}}$.
 $\underline{\hspace{2cm}} \sqrt{0.875} \approx 0.9354, \sqrt[4]{0.875} \approx 0.9672,$
sum ≈ 2.87 .

Entire Radicals and Mixed Radicals — Part One

- Without a calculator, arrange in order from *greatest to least*:
 $3\sqrt{5}$, $5\sqrt{3}$, $\sqrt{15}$, $2\sqrt{8}$, $8\sqrt{2}$.
 $8\sqrt{2}$ (11.31) > $5\sqrt{3}$ (8.66) > $3\sqrt{5}$ (6.71) > $2\sqrt{8} = 4\sqrt{2}$ (5.66) > $\sqrt{15}$ (3.87).
- Two students find the hypotenuse PQ of a right triangle with legs $\sqrt{34}$ and $\sqrt{38}$. Louis rounds each leg; Asia simplifies radicals first.
 - Compute each to the nearest hundredth.
Louis ≈ 8.48 ; *Asia* $6\sqrt{2} \approx 8.49$.
 - Which is more accurate? *Asia (kept exact form before rounding)*.
 - Exact mixed radical. $6\sqrt{2}$.
- Convert each to a *mixed radical* (simplest form).
 - $\sqrt{96}$ $4\sqrt{6}$
 - $\sqrt{242}$ $11\sqrt{2}$
 - $\frac{2}{3}\sqrt{180}$ $4\sqrt{5}$
 - $\frac{1}{8}\sqrt{320}$ $\sqrt{5}$
 - $\sqrt{245}$ $7\sqrt{5}$
 - $4\sqrt{338}$ $52\sqrt{2}$
 - $\sqrt{1250}$ $25\sqrt{2}$
 - $\sqrt{66}$ *cannot convert (no perfect-square factor > 1)*
 - $-\frac{5}{6}\sqrt{304} - \frac{10}{3}\sqrt{19}$
 - $\sqrt{980}$ $14\sqrt{5}$
 - $4\sqrt{272}$ $16\sqrt{17}$
 - $-3\sqrt{288} - 36\sqrt{2}$
 - $2\sqrt{369}$ $6\sqrt{41}$
 - $\sqrt{364}$ $2\sqrt{91}$
 - $\frac{2}{5}\sqrt{450}$ $6\sqrt{2}$
 - $\frac{7}{11}\sqrt{341}$ *cannot convert (341 = 11 · 31)*
- Convert to a *mixed radical* where the radicand is a whole number.
 - $\sqrt{\frac{2}{9}}$ $\frac{1}{3}\sqrt{2}$
 - $\sqrt{\frac{5}{4}}$ $\frac{\sqrt{5}}{2}$
 - $\sqrt{\frac{18}{25}}$ $\frac{3\sqrt{2}}{5}$
 - $7\sqrt{\frac{20}{49}}$ $2\sqrt{5}$
- Convert to *entire radical* form.
 - $2\sqrt{6}$ $\sqrt{24}$
 - $3\sqrt{7}$ $\sqrt{63}$
 - $5\sqrt{15}$ $\sqrt{375}$
 - $12\sqrt{2}$ $\sqrt{288}$
 - $3\sqrt{25}$ $\sqrt{225}$
 - $-8\sqrt{3} - \sqrt{192}$
 - $9\sqrt{10}$ $\sqrt{810}$
 - $-4\sqrt{5} - \sqrt{80}$
- Convert the following to *entire radical* form.
 - $\frac{1}{3}\sqrt{27}$ $\sqrt{3}$
 - $15\sqrt{225}$
 - $\frac{3}{2}\sqrt{8}$ $\sqrt{18}$
 - $3^2\sqrt{21}$ $\sqrt{1701}$
- Given $\sqrt{6} \approx 2.45$ and $\sqrt{60} \approx 7.75$, approximate:
 - $\sqrt{600}$ 24.5
 - $\sqrt{6000}$ 77.5
 - $\sqrt{600000}$ 775
 - $\sqrt{0.06}$ 0.245
 - $\sqrt{0.6}$ 0.775
 - $\sqrt{24}$ 4.90
 - $\sqrt{540}$ 23.25
 - $\sqrt{\frac{6}{25}}$ 0.49
- Arrange from *greatest to least*:
 $3\sqrt{7}$, $5\sqrt{3}$, $\sqrt{60}$, $2\sqrt{11}$, $\frac{1}{2}\sqrt{200}$.
 $5\sqrt{3}$ (8.66) > $3\sqrt{7}$ (7.94) > $\sqrt{60}$ (7.75) > $\frac{1}{2}\sqrt{200}$ (7.07) > $2\sqrt{11}$ (6.63).
- In right $\triangle XYZ$ with legs 19 cm and 5 cm, find hypotenuse XY :
 - entire radical $\sqrt{386}$
 - mixed radical (*already simplest*)
 - decimal (nearest hundredth) 19.65
- Find the missing side in simplest mixed radical form.
 - legs 4, 8; hypotenuse x $x = \sqrt{80} = 4\sqrt{5}$
 - legs 5, 6; hypotenuse x $x = \sqrt{61}$
 - hypotenuse 8, leg 6; other leg x $x = \sqrt{28} = 2\sqrt{7}$

11. The length of \overline{KL} for legs $\sqrt{6}$ and $\sqrt{24}$ is
A. $\sqrt{540}$ B. $3\sqrt{2}$ C. $\sqrt{30}$ D. $9\sqrt{2}$ *Correct: C.*
12. Without a calculator, which radical is *not* equal to the others?
A. $12\sqrt{2}$ B. $\sqrt{288}$ C. $6\sqrt{8}$ D. $4\sqrt{72}$ *D. ($\sqrt{288} = 12\sqrt{2}$, $6\sqrt{8} = 12\sqrt{2}$, but $4\sqrt{72} = 24\sqrt{2}$.)*
13. On a clear day, $d = \sqrt{13h}$ (km), where h metres is eye level above ground. From a 698.2 m building with eye level 1.8 m above the roof, write $d = a\sqrt{b}$ and find $a + b$. *$h = 700$, $d = \sqrt{9100} = 10\sqrt{91}$; $a + b = 101$.*
14. Using Heron's formula, a triangle with sides 14, 15, 25 has area $A = p\sqrt{26}$. Find p . *$p = 18$.*
15. A square of side 8 cm is inscribed in a larger square by joining midpoints. If larger side is $p\sqrt{q}$, find pq . *$8\sqrt{2} \Rightarrow pq = 16$.*

Entire Radicals and Mixed Radicals — Part Two

- Convert the following radicals to mixed radicals in simplest form.
 - $\sqrt[3]{48} \ 2\sqrt[3]{6}$
 - $\sqrt[3]{128} \ 4\sqrt[3]{2}$
 - $\sqrt[3]{2000} \ 10\sqrt[3]{2}$
 - $5\sqrt[3]{-81} \ -15\sqrt[3]{3}$
 - $\frac{5}{6}\sqrt[3]{108} \ \frac{5}{2}\sqrt[3]{4}$
 - $5\sqrt[4]{162} \ 15\sqrt[4]{2}$
 - $5\sqrt[3]{192} \ 40\sqrt[3]{3}$
 - $-2\sqrt[3]{625} \ -10\sqrt[3]{5}$
- Convert the following mixed radicals to entire radicals.
 - $2\sqrt[5]{2} \ \sqrt[5]{32}$
 - $3\sqrt[3]{4} \ \sqrt[3]{108}$
 - $-3\sqrt[4]{3} \ -\sqrt[4]{243}$
 - $-10\sqrt[3]{5} \ -\sqrt[3]{5000}$
 - $2\sqrt[5]{6} \ \sqrt[5]{192}$
 - $\frac{1}{2}\sqrt[3]{16} \ \sqrt[3]{2}$
 - $\frac{3}{10}\sqrt[4]{100000} \ \sqrt[4]{810}$
 - $-5\sqrt[3]{9} \ -\sqrt[3]{1125}$
- Arrange, least to greatest (no calculator):
 $7\sqrt[6]{1}, -3\sqrt[3]{-27}, \frac{5}{2}\sqrt[4]{16}, 3\sqrt[3]{64}.$
 $-3\sqrt[3]{-27} < 7\sqrt[6]{1} < \frac{5}{2}\sqrt[4]{16} < 3\sqrt[3]{64}.$
- Consider $2\sqrt[3]{11}, 3\sqrt[3]{3}, 4\sqrt[3]{2}, 2\sqrt[3]{6}.$
 - Explain how to compare without a calculator. *Cube each expression; compare the resulting values.*
 - Order least to greatest.
 $2\sqrt[3]{6} < 3\sqrt[3]{3} < 4\sqrt[3]{2} < 2\sqrt[3]{11}.$
- $\sqrt[3]{240}$ is equivalent to
A. $2\sqrt[3]{40}$ **B.** $4\sqrt[3]{15}$ **C.** $2\sqrt[3]{30}$ **D.** $8\sqrt[3]{30}$
Correct: B.
- Consider the statements:
 - $-3\sqrt[4]{8} = 3\sqrt[4]{-8},$
 - $-2\sqrt[3]{7} = 2\sqrt[3]{-7}.$**A.** Both true **B.** Both false **C.** 1 true, 2 false
D. 1 false, 2 true *Correct: D.*
- The mixed radical $\frac{1}{12}\sqrt[3]{128}$ equals $a\sqrt[3]{b}$ in simplest form. Find $a + b$ (nearest tenth). $\frac{1}{3}\sqrt[3]{2} \Rightarrow 2.3.$
- $\sqrt{3x} \cdot \sqrt{2x}$ is equivalent to
A. $\sqrt{6x}$ **B.** $\sqrt{36x^2}$ **C.** $6\sqrt{x}$ **D.** $x\sqrt{6}$
 $\sqrt{3x}\sqrt{2x} = \sqrt{6x^2} = x\sqrt{6} \ (x \geq 0).$
Answer: D.
- Express as an *entire* radical.
 - $6\sqrt{y} \ \sqrt{36y}$
 - $8\sqrt{c^2} \ \sqrt{64c^2}$
 - $10\sqrt{2yz^3} \ \sqrt{200yz^3}$
 - $-3\sqrt[3]{x^2} \ \sqrt[3]{-27x^2}$
 - $c\sqrt{c} \ \sqrt{c^3}$
 - $x\sqrt{3y^3} \ \sqrt{3x^2y^3}$
 - $11c^2\sqrt{c^2d} \ \sqrt{121c^6d}$
 - $5a^3b\sqrt{3a^2b} \ \sqrt{75a^8b^3}$
- Express as a mixed radical in simplest form.
 - $\sqrt{a^5} \ a^2\sqrt{a}$
 - $\sqrt{t^3} \ t\sqrt{t}$
 - $\sqrt{x^{11}} \ x^5\sqrt{x}$
 - $\sqrt[3]{x^4} \ x\sqrt[3]{x}$
 - $\sqrt[3]{b^8} \ b^2\sqrt[3]{b^2}$
 - $\sqrt[4]{x^6} \ x\sqrt[4]{x^2}$
- Express as a mixed radical.
 - $\sqrt{8y^2} \ 2y\sqrt{2}$
 - $\sqrt{16p^3} \ 4p\sqrt{p}$
 - $\sqrt{75y^3z^4} \ 5yz^2\sqrt{3y}$
 - $\sqrt{300a^9w^7} \ 30a^4w^3\sqrt{3aw}$
 - $5\sqrt{28c^4d^3} \ 10c^2d\sqrt{7d}$
 - $-6\sqrt{29a^4b^8} \ -6a^2b^4\sqrt{29}$

Quick Check

- Which is not a prime factor of 14014?
A. 7 B. 11 C. 13 D. 17 **D**
- How many numbers in the list 7, 11, 17, 21 are prime factors of 3234?
A. 1 B. 2 C. 3 D. 4 **B**
- The sum of prime factors of 160797 is _____.
73
- The GCF of 6699 and 8265 is _____. **87**
- LCM of 14 and 105 equals GCF of P, Q . Which must be false?
A. P multiple of 7
B. Q multiple of 21
C. $P < 200$
D. $Q > 2000$ **C**
- If x is a perfect square, minimum value of d (in the given factor tree) is
A. 2 B. 3 C. 6 D. 9 **D**
- If x is a perfect cube, the minimum value of x is _____. **216**
- \otimes is irrational. Its decimal representation is
A. terminating & repeating
B. terminating & non-repeating
C. non-terminating & repeating
D. non-terminating & non-repeating **D**
- Which are rational? (I) 1.0100100001... (non-repeating), (II) $\sqrt[3]{\frac{8}{27}}$, (III) 0.04, (IV) 0.29
A. III, IV
B. II, III, IV
C. I, II, III, IV
D. Other **B**
- The rational number $1.\overline{54}$ as $\frac{c}{d}$; the value of c is
A. 17 B. 11 C. 6 D. 4 **A**
- M, N irrationals with $30 < M < 40, 3 < N < 4$. The value of $\sqrt{M} + \sqrt{N}$ is best represented by
A. P B. Q C. R D. S **B**
- Largest among $\sqrt[3]{67}, \sqrt[4]{98}, \sqrt{19}, \sqrt[5]{201}$ is
A. $\sqrt{19}$ B. $\sqrt[4]{98}$ C. $\sqrt[3]{67}$ D. $\sqrt[5]{201}$ **A**
- The length $12\sqrt[4]{4000}$ m has index and radicand
A. 4 and 4000
B. 12 and 4000
C. 4 and 12
D. 4000 and 4 **A**
- When $7\sqrt[3]{6}$ is written as an entire radical, the radicand is _____. **2058**
- Three students rewrote $\sqrt{4050}$. Who is correct?
A. I only
B. II and III
C. All three
D. Other **B**
- Circle area: if area $120\pi \text{ cm}^2$, radius is
A. 60 B. $12\sqrt{10}$ C. $2\sqrt{30}$ D. $2\sqrt{15}$ **C**
- A cube with volume 720 mm^3 has edge length $a\sqrt[3]{b}$ mm. Find $a + b$. **92**
- Consider $4\sqrt[3]{3}, 5\sqrt{x}, 16\sqrt{y}$. Which is correct?
A. $x < y < z$
B. $z < x < y$
C. $y < z < x$
D. $z < y < x$ **C**