Math 10 — Unit 1 Quick Check

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Instructions. Answer each question. For numeric response, write your final value clearly in the box.

A. Multiple Choice

Select *one* option.

1. Which of the following is a prime number?

(A) 204 (B) 221 (C) 225 (D) 199

Solution: (D) 199

2. The prime factorization of 360 is

(A) $2^3 \cdot 3^2 \cdot 5$ (B) $2 \cdot 3^3 \cdot 5^2$ (C) $2^2 \cdot 3^2 \cdot 5^2$ (D) $2^4 \cdot 3 \cdot 5$

Solution: (A) $2^3 \cdot 3^2 \cdot 5$

3. gcd(108, 252) equals

(A) 12 (B) 18 (C) 24 (D) 36

Solution: (D) 36

4. lcm(12, 18) equals

(A) 30 (B) 36 (C) 216 (D) 1

Solution: (B) 36

5. Which is a perfect square?

(A) $2^3 \cdot 3^2 \cdot 5$ (B) $2^4 \cdot 3^2 \cdot 5^2$ (C) $2^3 \cdot 3^3 \cdot 5$ (D) $2^5 \cdot 3^2 \cdot 5$

Solution: (B) $2^4 \cdot 3^2 \cdot 5^2$

6. Which is a perfect cube?

(A) $2^4 \cdot 3^3 \cdot 5^2$ (B) $2^6 \cdot 3^5 \cdot 5^3$ (C) $2^3 \cdot 3^6 \cdot 5^3$ (D) $2^2 \cdot 3^3 \cdot 5^4$

Solution: (C) $2^3 \cdot 3^6 \cdot 5^3$

7. Write $5\sqrt{7}$ as an entire radical.

(A) $\sqrt{35}$ (B) $\sqrt{350}$ (C) $\sqrt{175}$ (D) $\sqrt{105}$

Solution: (C) $\sqrt{175}$

8. Convert to a *simplified* mixed radical: $\sqrt{72}$.

(A) $\sqrt{36}$ (B) $6\sqrt{2}$ (C) $3\sqrt{8}$ (D) $\sqrt{72}$

Solution: (B) $6\sqrt{2}$

9. Simplify $\sqrt{45\,x^3y^2}$ for $x, y \geq 0$.

(A) $xy\sqrt{45}$ (B) $3xy\sqrt{5x}$ (C) $x\sqrt{45y}$ (D) $\sqrt{45}\,xy$

Solution: (B) $3xy\sqrt{5x}$

10. $\sqrt[3]{-512}$ equals

(A) -9 (B) 9 (C) -7 (D) -8

Solution: (D) -8

11. Which number is **irrational**?

(A) $\sqrt{10}$ (B) $0.\overline{27}$ (C) $\frac{3}{7}$ (D) 4.125

Solution: (A) $\sqrt{10}$

12. Simplify $\sqrt{12} \cdot \sqrt{18}$.

(A) $\sqrt{30}$ (B) $6\sqrt{6}$ (C) $\sqrt{216}$ (D) $12\sqrt{18}$

Solution: (B) $6\sqrt{6}$

13. Which of the following is a perfect fourth power?

(A) 27 (B) 81 (C) 80 (D) 82

Solution: (B) 81

B. Numeric Response

Write your final answer clearly in the box.

1. Compute gcd(108, 252).

Solution: 36

2. Compute lcm(12, 18, 20).

Solution: 180

3. Let the prime factorization of 504 be $2^a \cdot 3^b \cdot 5^c \cdot 7^d$. Compute $a + b + c + d$.

Solution: 6

C. Written Response

Show full reasoning; express final answers with positive exponents/radicals.

1. Simplify $\sqrt{108\,x^5y^3}$ for $x, y \geq 0$.

Solution: $6x^2y\sqrt{3xy}$

2. Rationalize and simplify: $\dfrac{10}{\sqrt{18}}$.

Solution: $\dfrac{5\sqrt{2}}{3}$

3. Convert $4.\overline{27}$ to a fraction in simplest form.

Solution: $\dfrac{47}{11}$

4. Determine gcd and lcm of 180 and 168.

Solution: $180 = 2^2 \cdot 3^2 \cdot 5,\ 168 = 2^3 \cdot 3 \cdot 7$. Thus gcd = 12, lcm = 2520.

5. Let $N = 2^1 \cdot 3^2 \cdot 5^0 \cdot 7^2$. Decide whether N is a perfect square and/or a perfect cube. If not a cube, find the least positive integer m so that Nm is a perfect cube.

Solution: Not a square (odd exponent on 2); not a cube. Least $m = 2^2 \cdot 3^1 \cdot 7^1 = 84$.

6. Simplify and write with radicals (no negative exponents):

$$\frac{\sqrt{24x^3y^2} \cdot \sqrt{6xy}}{\sqrt{3x}} \quad (x, y \geq 0).$$

Solution: $4xy\sqrt{3xy}$

7. Find the sum of all integers between 120 and 360 inclusive that are multiples of 2 or 3.

Solution: 38640

8. Using two iterations of the Babylonian method with $x_0 = 8$, approximate $\sqrt{73}$ correct to four decimal places.

Solution: $x_1 = 8.562500000,\ x_2 = 8.544023723$, hence $\sqrt{73} \approx 8.5440$.

9. For each value, state all sets it belongs to among $\mathbb{N}, \mathbb{W}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$:

i. $a = \dfrac{3}{7}$

ii. $b = 2.375$

iii. $c = -6$

iv. $d = 5$

v. $e = \sqrt{5}$

Solution: $a \in \mathbb{Q}, \mathbb{R}$; $b \in \mathbb{Q}, \mathbb{R}$; $c \in \mathbb{Z}, \mathbb{Q}, \mathbb{R}$; $d \in \mathbb{N}, \mathbb{W}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$; $e \in \mathbb{R}$ only.

10. Find the number of positive divisors of 840.

Solution: $840 = 2^3 \cdot 3 \cdot 5 \cdot 7 \Rightarrow (3+1)(1+1)^3 = 32$.

11. How many positive integers less than 100 are multiples of 3 or 4 but not 5?

Solution: 40

12. Find all ordered pairs (m, n) of positive integers such that $\gcd(m, n) = 18$ and $\operatorname{lcm}(m, n) = 540$.

Solution: We have $m \cdot n = \gcd(m, n) \cdot \operatorname{lcm}(m, n) = 9720$. Then $(m/18)(n/18) = 9720/324 = 30$. So $x = \frac{m}{18}, y = \frac{n}{18}$ satisfy $xy = 30$ and $\gcd(x, y) = 1$. Coprime factor pairs of 30 are $(1, 30), (30, 1), (2, 15), (15, 2), (3, 10), (10, 3), (5, 6), (6, 5)$, giving 8 solutions. Example: $(m, n) = (18, 540), (540, 18), (36, 270), (270, 36), (54, 180), (180, 54), (90, 108), (108, 90)$.

13. $A = \{\, 5k \mid k \in \mathbb{Z}^+,\ 5k \leq 100 \,\}$ $\quad B = \{\, 2k \mid k \in \mathbb{Z}^+,\ 2k \leq 100 \,\}$ $\quad C = \{\, 3k \mid k \in \mathbb{Z}^+,\ 3k \leq 100 \,\}$.

(i) $\sum A$

(ii) $\sum (A \cap B)$

(iii) $\sum (A \cup B)$

(iv) $|(B \cap C) \setminus A|$

(v) $\sum (B \setminus (A \cup C))$

Solution: (i) 1050; (ii) 550; (iii) 3050; (iv) 13; (v) 1364.

14. List all positive divisors of 120. Then determine:

(i) the probability that a randomly chosen divisor is even;

(ii) the probability that, with replacement, two randomly chosen divisors are both multiples of 4;

(iii) the probability that, with replacement, at least one of two randomly chosen divisors is a multiple of 2;

(iv) the probability that, without replacement, two randomly chosen divisors are multiples of 2 or 3;

Solution: Divisors: $1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120$. (i) $12/16 = 3/4$; (ii) $(8/16)^2 = 1/4$; (iii) $1 - (4/16)^2 = 15/16$; (iv) $91/120$.

15. Prove that $\sqrt{2}$ is irrational.

Solution: Suppose $\sqrt{2} = \frac{p}{q}$ in lowest terms. Then $p^2 = 2q^2$. Hence p is even, so $p = 2k$. Then $4k^2 = 2q^2 \implies q^2 = 2k^2$, so q is even. Both p, q even contradicts lowest terms. Therefore $\sqrt{2}$ is irrational.

16. Find all ordered pairs (m, n) of positive integers such that $\gcd(m, n) = 18$ and $\operatorname{lcm}(m, n) = 540$.

Solution: We have $m \cdot n = \gcd(m, n) \cdot \operatorname{lcm}(m, n) = 9720$. Then $(m/18)(n/18) = 9720/324 = 30$. So $x = \frac{m}{18}, y = \frac{n}{18}$ satisfy $xy = 30$ and $\gcd(x, y) = 1$. Coprime factor pairs of 30 are $(1, 30), (30, 1), (2, 15), (15, 2), (3, 10), (10, 3), (5, 6), (6, 5)$, giving 8 solutions. Example: $(m, n) = (18, 540), (540, 18), (36, 270), (270, 36), (54, 180), (180, 54), (90, 108), (108, 90)$.

17. Find the last digit of 7^{2025}.

Solution: The last digits of powers of 7 cycle: $7, 9, 3, 1$. Since $2025 \equiv 1 \pmod 4$, the last digit is 7.

18. Determine the number of trailing zeros in 2025!.

Solution: Trailing zeros count is $\left\lfloor \frac{2025}{5} \right\rfloor + \left\lfloor \frac{2025}{25} \right\rfloor + \left\lfloor \frac{2025}{125} \right\rfloor + \left\lfloor \frac{2025}{625} \right\rfloor = 405 + 81 + 16 + 3 = 505$.

19. Show that $\sqrt[3]{2}$ is irrational.

Solution: Suppose $\sqrt[3]{2} = \frac{p}{q}$ in lowest terms. Then $p^3 = 2q^3$. So p is even, $p = 2k$. Then $8k^3 = 2q^3 \implies q^3 = 4k^3 \implies q$ even. Contradiction. Thus irrational.

20. $\star$ Compute $\displaystyle\sum_{d|n} d$ for $n = 2^4 \cdot 3^3 \cdot 5^2 \cdot 7$.

Solution: Use $\sigma(n) = \prod \frac{p^{e+1}-1}{p-1}$. Here $n = 2^4 \cdot 3^3 \cdot 5^2 \cdot 7$. So $(2^5 - 1)/(1) = 31$, $(3^4 - 1)/(2) = 40$, $(5^3 - 1)/(4) = 31$, $(7^2 - 1)/(6) = 8$. Product $31 \cdot 40 \cdot 31 \cdot 8 = 307{,}520$.

D. True / False

Decide if each statement is true or false (use formal set notation).

1. $7 \in \mathbb{N}$

Solution: True

2. $-3 \in \mathbb{W}$

Solution: False

3. $\dfrac{5}{8} \in \mathbb{Q}$

Solution: True

4. $0.\overline{3} \in \mathbb{Q}$

Solution: True

5. $\sqrt{6} \in \mathbb{Q}$

Solution: False