Extra Practice

Math 10 · Mr. Merrick · October 1, 2025

1. $12^{2x+1} = 2^{3x+7} \cdot 3^{3x-4}$ [Write $12 = (2^2 \cdot 3)$. Then $12^{2x+1} = 2^{4x+2}\, 3^{\,2x+1}$. Match prime exponents with RHS $2^{3x+7}\, 3^{\,3x-4}$: $4x+2 = 3x+7 \Rightarrow x = 5$ and $2x+1 = 3x-4 \Rightarrow x = 5$.]

2. If $x^3y^5 = 2^{11}3^{13}$ and $\dfrac{x}{y^2} = \dfrac{1}{27}$, find x and y. [Let $x = 2^a3^b$, $y = 2^c3^d$. Then $\frac{x}{y^2} = 3^{-3}$ gives $a - 2c = 0$, $b - 2d = -3$. Also x^3y^5 gives $3a + 5c = 11$, $3b + 5d = 13$. Solve: from $a = 2c$, so $6c + 5c = 11 \Rightarrow c = 1$, hence $a = 2$. From $b = 2d - 3$ and $3(2d-3) + 5d = 13 \Rightarrow 11d = 22 \Rightarrow d = 2$, so $b = 1$. Thus $x = 2^2 \cdot 3 = 12$, $y = 2 \cdot 3^2 = 18$.]

3. $y = ax^r$ passes through $(2, 1)$ and $(32, 4)$. Find r. [$\dfrac{1}{4} = \dfrac{a2^{\,r}}{a32^{\,r}} = (\frac{1}{16})^r$. So $16^r = 4 = 2^2 \Rightarrow 2^{4r} = 2^2 \Rightarrow r = \frac{1}{2}$.]

4. Solve for x and y: $2^{x+3} + 2^x = 3^{y+2} - 3^y$ [Factor: $2^x(8+1) = 3^y(9-1) \Rightarrow 9 \cdot 2^x = 8 \cdot 3^y$. Writing $9 = 3^2$, $8 = 2^3$, we get $2^{x-3}\, 3^2 = 3^y$. Hence $2^{x-3} = 1 \Rightarrow x = 3$ and then $3^2 = 3^y \Rightarrow y = 2$.]

5. If $f(x) = 2^{4x-2}$, find $f(x) \cdot f(1-x)$ in simplest form. [$f(1-x) = 2^{4(1-x)-2} = 2^{2-4x}$. Product $= 2^{(4x-2)+(2-4x)} = 2^0 = 1$.]

6. Solve for x: $3^{x+2} + 2^{x+2} + 2^x = 2^{x+5} + 3^x$ [Group: $(3^{x+2} - 3^x) = (2^{x+5} - (2^{x+2} + 2^x))$. That is $3^x(9-1) = 2^x(32-4-1)$, i.e. $8 \cdot 3^x = 27 \cdot 2^x$. Thus $(\frac{3}{2})^x = \frac{27}{8} = (\frac{3}{2})^3 \Rightarrow x = 3$.]

7. Solve for x: $5^{\,x-1} = 125 \cdot 25^x$ [Convert to base 5: RHS $= 5^3 \cdot (5^2)^x = 5^{2x+3}$. So $5^{x-1} = 5^{2x+3} \Rightarrow x - 1 = 2x + 3 \Rightarrow x = -4$.]

8. If $p^2q^3 = 2^6 \cdot 3^9$ and $\dfrac{p}{q} = 6$, find p and q. Answer in positive index form. [No integer solution. Let $p = 2^a 3^b$, $q = 2^c 3^d$. From $p/q = 2 \cdot 3$ we get $a - c = 1$, $b - d = 1$. From p^2q^3 we get $2a + 3c = 6$, $2b + 3d = 9$. Solve to get $c = \frac{4}{5}$, $d = \frac{7}{5}$, so $q = 2^{4/5}3^{7/5}$ and $p = 6q = 2^{9/5}3^{12/5}$.]

9. The function $y = k \cdot a^x$ passes through $(0, 3)$ and $(2, 12)$. Find a. [From $(0, 3)$, $k = 3$. Then $12 = 3a^2 \Rightarrow a^2 = 4 \Rightarrow a = 2$ (positive base).]

10. $7^{m+1} - 7^m = 6 \cdot 7^2$. Solve for m. [Factor $7^m(7 - 1) = 6 \cdot 49 \Rightarrow 6 \cdot 7^m = 294 \Rightarrow 7^m = 49 \Rightarrow m = 2$.]

11. If $g(x) = 3^{2x+1}$, compute $\dfrac{g(x+1)}{g(x-1)}$. [$\dfrac{3^{2(x+1)+1}}{3^{2(x-1)+1}} = 3^{(2x+3)-(2x-1)} = 3^4 = 81$.]

12. Solve for integers x, y: $2^x + 2^y = 10$. [WLOG $x \leq y$. Then $2^x(1 + 2^{y-x}) = 10 = 2 \cdot 5$. Hence $2^x = 2 \Rightarrow x = 1$ and $1 + 2^{y-1} = 5 \Rightarrow 2^{y-1} = 4 \Rightarrow y = 3$. Also the symmetric pair $(3, 1)$.]

13. If $h(x) = \dfrac{4^x + 2^{2x}}{8^x}$, simplify $h(x)$. [$4^x = 2^{2x}$, so numerator $= 2^{2x} + 2^{2x} = 2^{2x+1}$. Divide by 2^{3x} to get 2^{1-x}.]

14. Solve for x: $9^{x-1} \cdot 3^{2x} = 81$. [$9 = 3^2$, $81 = 3^4$. LHS $= 3^{2(x-1)} \cdot 3^{2x} = 3^{4x-2}$. Set exponents equal: $4x - 2 = 4 \Rightarrow x = \frac{3}{2}$.]

15. If $y = ab^x$ passes through $(1, 6)$ and $(3, 54)$, find a and b. [Divide: $\frac{54}{6} = b^{3-1} \Rightarrow b^2 = 9 \Rightarrow b = 3$ (positive). Then $a = 6/b = 2$.]

16. Suppose $2^p = 3^q$. Express p in terms of q using logarithms. [Take $\log_2$: $p = \log_2(3^q) = q \log_2 3$.]

17. Solve for x: $4^{x+2} = 2^{3x}$. [$4 = 2^2 \Rightarrow 2^{2x+4} = 2^{3x} \Rightarrow 2x + 4 = 3x \Rightarrow x = 4$.]

18. If $M = 2^5 3^4$ and $N = 2^2 3^6$, find $\dfrac{M}{N}$ in simplest form. [$\dfrac{2^5 3^4}{2^2 3^6} = 2^3 3^{-2} = \dfrac{8}{9}$.]

19. Evaluate $\left(27\right)^{-2(3^{-1})}$. [$27^{-2/3} = (3^3)^{-2/3} = 3^{-2} = \frac{1}{9}$.]

20. Solve for x: $\left(\dfrac{5}{8}\right)^x \left(\dfrac{25}{64}\right)^2 = \dfrac{5}{8}$ [$\dfrac{25}{64} = (\frac{5}{8})^2 \Rightarrow \text{LHS} = (\frac{5}{8})^{x+4} = (\frac{5}{8})^1 \Rightarrow x + 4 = 1 \Rightarrow x = -3$.]

21. Order 4^{40}, 3^{50}, 2^{80} from least to greatest. [$4^{40} = (2^2)^{40} = 2^{80}$, and $\dfrac{2^{80}}{3^{50}} = (\frac{256}{243})^{10} > 1$. Hence $3^{50} < 4^{40} = 2^{80}$.]

22. If $a = 9^{12}$ and $b = 12^9$, which is larger? [$\dfrac{a}{b} = \dfrac{3^{24}}{(3\cdot 4)^9} = \dfrac{3^{15}}{4^9} > 1$ (e.g., $(\frac{243}{64})^3 > 1$), so $a > b$.]

23. Evaluate $\left(81\right)^{-3(4^{-1})}$. [$81^{-3/4} = (3^4)^{-3/4} = 3^{-3} = \frac{1}{27}$.]

24. Solve for x: $\left(\dfrac{7}{9}\right)^x \left(\dfrac{49}{81}\right)^3 = \dfrac{7}{9}$ [$\dfrac{49}{81} = (\frac{7}{9})^2 \Rightarrow (\frac{7}{9})^{x+6} = (\frac{7}{9})^1 \Rightarrow x = -5$.]

25. Order 6^{20}, 3^{30}, 2^{60} from least to greatest. [$\dfrac{2^{60}}{6^{20}} = (\frac{4}{3})^{20} > 1$ so 2^{60} is largest; also $\dfrac{6^{20}}{3^{30}} = \dfrac{2^{20}}{3^{10}} > 1$, so $3^{30} < 6^{20} < 2^{60}$.]

26. Evaluate $\left(125\right)^{-4(3^{-1})}$. [$125^{-4/3} = (5^3)^{-4/3} = 5^{-4} = \frac{1}{625}$.]

27. Which is larger: 7^{12} or 14^9? [$\dfrac{7^{12}}{14^9} = \dfrac{7^{12}}{2^9 7^9} = \dfrac{7^3}{2^9} = \frac{343}{512} < 1 \Rightarrow 14^9 > 7^{12}$.]

28. $(Assume \quad a, b \neq 0.)$ Write the expression as $\dfrac{a^m}{b^n}$ and find $m+n$:

$$\frac{\left(a^2b^{-3}\right)^{-2}\left(\frac{a^5}{b^2}\right)^3}{\left(ab^{-1}\right)^{-4}/\left(a^{-3}b^2\right)}$$

$[\,m+n=14\,]$

29. $(Assume \quad a, b \neq 0.)$ Write as $\dfrac{a^p}{b^q}$ and determine $p+q$:

$$\frac{\left(a^{-1}b^2\right)^3\left(\frac{a^4}{b}\right)^{-2}}{\left(a^2b\right)^{-1}\left(\frac{a}{b^3}\right)^2}$$

$[\,p+q=-26\,]$

30. $(Assume \quad a, b \neq 0.)$ Express in the form $\dfrac{a^r}{b^s}$ and compute $r+s$:

$$\left[\frac{(ab^2)^{-3}\left(\frac{a^{-2}}{b}\right)^4}{(a^3b^{-1})^{-2}}\right]^2$$

$[\,r+s=14\,]$

31. $(Assume \quad a, b \neq 0.)$ Express as $\dfrac{a^u}{b^v}$ and find $u+v$:

$$\frac{(a^{-2}b)^5}{(ab^{-3})^{-2}} \cdot \frac{\left(\frac{a^3}{b^2}\right)^{-1}}{(a^{-1}b^4)^2}$$

$[\,u+v=-2\,]$

32. ($Assume\ \ a, b \neq 0.$) Write as $\dfrac{a^\alpha}{b^\beta}$ and evaluate $\alpha + \beta$:

$$\left[\frac{(a^{1/2}b^{-3/2})^{-4}}{(a^{-3/2}b^{1/2})^2}\right]\left(\frac{a}{b}\right)^{-3}$$

[$\alpha + \beta = -10$]

33. ($Assume\ \ a, b \neq 0.$) Write as $\dfrac{a^h}{b^k}$ and find $h + k$:

$$\left(\frac{a^{-2}b^5}{a^3b^{-1}}\right)^{-3} \cdot \frac{(ab)^4}{(a^{-1}b)^{-2}}$$

[$h + k = 29$]

34. ($Assume\ \ a, b, c \neq 0.$) Write the expression in the form $\dfrac{a^\ell}{b^m c^n}$ and find $\ell + m + n$:

$$\frac{(a^3b^{-2}c)^{1/2}\,(a^{-1}b\,c^2)^{-3}}{(a^{1/2}b^{-3/2}c^{-1}){-2}}$$

[Result $= \dfrac{a^{11/2}}{b^7c^{15/2}}$, so $\ell + m + n = \frac{11}{2} + 7 + \frac{15}{2} = 20.$]

35. ($Assume\ \ b, c, d \neq 0.$) Write the expression in the form $\dfrac{b^s}{c^t d^u}$ and find $s + t + u$:

$$\frac{\left(\frac{d^2}{b^3c}\right)^{-2}\left(d\,b^{-1}c^{-2}\right)^5}{\left(\frac{bc^2}{d^3}\right)^{-3}(d^{-2}b\,c^{-1})^{-4}}$$

[Result $= \dfrac{b^8}{c^6d^{16}}$, so $s + t + u = 8 + 6 + 16 = 30.$]