Factoring by Grouping

Math 10 · Mr. Merrick · October 29, 2025

Goal. Recognize when to factor expressions by grouping and apply it in several settings.

General plan (four terms). Split into two groups, factor each group, then factor out the *common binomial*. If grouping fails, try a different order or pull a GCF first.

GCF Warm-Up

Factor out the greatest common factor.

1) $12x^3 - 18x^2$
[$6x^2(2x - 3)$]

2) $28y^4 + 21y^3$
[$7y^3(4y + 3)$]

3) $9a^2b - 6ab^2$
[$3ab(3a - 2b)$]

4) $14m^2n^3 - 42mn^2$
[$14mn^2(mn - 3)$]

5) $25p^3q + 10pq$
[$5pq(5p^2 + 2)$]

6) $8r^5 - 12r^3$
[$4r^3(2r^2 - 3)$]

Classic Four-Term Grouping

Factor completely by grouping.

7) $ax + ay + bx + by$
[$(a + b)(x + y)$]

8) $3x^2 + 6x + 2x + 4$
[$(3x + 2)(x + 2)$]

9) $5m^2 - 15m + 2m - 6$
[$(5m + 2)(m - 3)$]

10) $4a^2 - 12a + 3a - 9$
[$(4a + 3)(a - 3)$]

11) $6u^2 + 9u - 4u - 6$
[$(2u + 3)(3u - 2)$]

12) $8y^2 - 10y - 4y + 5$
[$(4y - 5)(2y - 1)$]

13) $pq + 2p + q + 2$
[$(p + 1)(q + 2)$]

14) $r^2 + 5r + 2r + 10$
[$(r + 2)(r + 5)$]

15) $3x^2 - 2x - 6x + 4$
[$(3x - 2)(x - 2)$]

Rearrangement Before Grouping

Reorder terms (and/or factor GCFs) to make grouping work.

16) $2x - 6 + 3x^2 - 9x$
[$(3x^2 - 9x) + (2x - 6) = 3x(x - 3) + 2(x - 3) = (3x + 2)(x - 3)$]

17) $a^2 + 4a + 3b + 12$
[Does not factor further.]

18) $5y^2 + 10y + 4y + 8$
[$(5y^2 + 10y) + (4y + 8) = (5y + 4)(y + 2)$]

19) $6m^2 - 8m + 9n - 12$
[Does not factor further.]

20) $x^2 + 7x - 3x - 21$
[$(x + 7)(x - 3)$]

21) $9p^2 - 6pq + 3q^2 - 12$
[$3(3p^2 - 2pq + q^2 - 4)$ (no further integer factoring)]

Common-Binomial Pull-Out

Each group hides the same binomial factor.

22) $(x+y)(3x-1)+(x+y)(2)$
[$(x+y)(3x+1)$]

23) $4(a-b)^2-7(a-b)$
[$(a-b)(4(a-b)-7)$]

24) $(2m+n)^3-(2m+n)^2$
[$(2m+n)^2\big((2m+n)-1\big)$]

25) $k(k+5)-3(k+5)$
[$(k-3)(k+5)$]

26) $q(2q-1)+p(2q-1)$
[$(p+q)(2q-1)$]

27) $(u-v)(4u+1)-(u-v)(v+3)$
[$(u-v)(4u-v-2)$]

Variables Inside and Outside (careful GCFs)

28) $3x^2y-6xy+2xy-4y$
[$y(3x^2-6x+2x-4)=y(3x+2)(x-2)$]

29) $5a^2b+10ab^2-3ab-6b^2$
[$b(5a^2+10ab-3a-6b)=b(5a-3)(a+2b)$]

30) $7m^2n-14mn+5mn-10n$
[$n(7m^2-14m+5m-10)=n(7m+5)(m-2)$]

31) $4p^2q+6pq^2-8pq-12q$
[$2q(2p^2+3pq-4p-6)$]

32) $9x^2y-12xy+6xy-8y$
[$y(9x^2-6x-8)=y(3x-4)(3x+2)$]

33) $12r^2s-18rs+4rs-6s$
[$s(12r^2-14r-6)=2s(6r^2-7r-3)=2s(3r+1)(2r-3)$]

Decomposition Method (splitting the middle term)

For ax^2+bx+c, find two numbers that multiply to ac and add to b, *split* b, then group.

34) $x^2+7x+12$
[$(x+3)(x+4)$]

35) $2x^2+5x+2$
[$(2x+1)(x+2)$]

36) $3x^2-10x-8$
[$(3x+2)(x-4)$]

37) $6y^2+y-12$
[$(3y-4)(2y+3)$]

38) $4m^2-11m-3$
[$(4m+1)(m-3)$]

39) $5p^2+13p+6$
[$(5p+3)(p+2)$]

40) $7a^2-9a+2$
[$(7a-2)(a-1)$]

41) $8t^2-14t+3$
[$(4t-1)(2t-3)$]

42) $9z^2+12z+4$
[$(3z+2)^2$]

Three-and-Two Groupings & Standalone Grouping

Sometimes five terms (or more) still factor by grouping.

43) $3x^2 + 6x + 2x + 4 + 5(x + 2)$
[$(3x + 2)(x + 2) + 5(x + 2) = (x + 2)(3x + 7)$]

44) $a^2 + ab + b^2 - 4a - 4b - 4$
[No clean integer factorization over $\mathbb{Z}$.]

45) $pq + 2p + q + 2 + 3(p + 1)$
[$(p + 1)(q + 2) + 3(p + 1) = (p + 1)(q + 5)$]

46) $m^2 - 9 + 2m(n - 3)$
[Does not factor further.]

47) $x^3 + 2x^2 + 3x + 6$
[$x^2(x + 2) + 3(x + 2) = (x + 2)(x^2 + 3)$]

48) $4r^2 + 9 - 12r + 6(r - 1)$
[Does not factor further.]

Common Binomial "Hidden" by Signs

Watch for sign changes that still leave a common binomial.

49) $3(x - y) - 7(y - x)$
[$3(x - y) + 7(x - y) = 10(x - y)$]

50) $(a + 2b) - 4(2b + a)$
[$(a+2b) - 4(a+2b) = -3(a+2b)$]

51) $(2p-q)(r+s) - (q-2p)(r+s)$
[$(4p - 2q)(r + s) = 2(2p - q)(r + s)$]

52) $5(u + v) - 2(v + u)$
[$3(u + v)$]

53) $(m - n)^2 - (n - m)^2$
[0]

54) $k(1 - k) - (k - 1)$
[$-(k - 1)(k + 1)$]

Mixed Review

Factor completely:

55) $6x^2 + 19x + 10$
[$(3x + 2)(2x + 5)$]

56) $10a^2 - 11a - 6$
[$(5a + 2)(2a - 3)$]

57) $12m^2 + 11m - 5$
[$(3m - 1)(4m + 5)$]

58) $8p^3 + 4p^2 - 2p - 1$
[$(2p + 1)^2(2p - 1)$]

59) $9y^2 - 30y + 25$
[$(3y - 5)^2$]

60) $x^2 - 5x - 24$
[$(x - 8)(x + 3)$]

61) $2r^2 - 7r + 3$
[$(2r - 1)(r - 3)$]

62) $4t^3 + 6t^2 - 8t - 12$
[$2(2t + 3)(t^2 - 2)$]

63) $a^2b + ab^2 - 6a - 6b$
[$(a + b)(ab - 6)$]

64) $z^2 + z - 2 + 3(z + 2)$
[$(z + 2)^2$]