Factoring Word Problems

Math 10 · Mr. Merrick · November 17, 2025

Directions. Translate each problem into an equation and solve *by factoring*.

A. One-Number Quadratics

1. Find a number whose square is 16 more than 6 times the number.

   Solution. $x^2 = 6x + 16 \Rightarrow x^2 - 6x - 16 = 0 \Rightarrow (x-8)(x+2) = 0.$ $x = 8, -2.$

2. Find a number whose square is 30 less than 11 times the number.

   Solution. $x^2 = 11x - 30 \Rightarrow x^2 - 11x + 30 = 0 \Rightarrow (x-5)(x-6) = 0.$ $x = 5, 6.$

3. The square of a number is 18 more than 7 times the number.

   Solution. $x^2 = 7x + 18 \Rightarrow x^2 - 7x - 18 = 0 \Rightarrow (x-9)(x+2) = 0.$ $x = 9, -2.$

4. The sum of a number and its square is 56.

   Solution. $x^2 + x - 56 = 0 \Rightarrow (x+8)(x-7) = 0.$ $x = 7, -8.$

5. The product of a number and 9 more than itself is 136.

   Solution. $x(x+9) = 136 \Rightarrow x^2 + 9x - 136 = 0 \Rightarrow (x+17)(x-8) = 0.$ $x = 8, -17.$

6. A number *subtracted from its square* equals 72.

   Solution. $x^2 - x = 72 \Rightarrow x^2 - x - 72 = 0 \Rightarrow (x-9)(x+8) = 0.$ $x = 9, -8.$

7. Twice a number equals 3 less than its square.

   Solution. $2x = x^2 - 3 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x-3)(x+1) = 0.$ $x = 3, -1.$

8. The square of a number is 63 more than 2 times the number.

   Solution. $x^2 = 2x + 63 \Rightarrow x^2 - 2x - 63 = 0 \Rightarrow (x-9)(x+7) = 0.$ $x = 9, -7.$

9. The square of a number is 9 less than 10 times the number.

   Solution. $x^2 = 10x - 9 \Rightarrow x^2 - 10x + 9 = 0 \Rightarrow (x-1)(x-9) = 0.$ $x = 1, 9.$

10. The product of a number and 10 less than itself equals 56.

    Solution. $x(x-10) = 56 \Rightarrow x^2 - 10x - 56 = 0 \Rightarrow (x-14)(x+4) = 0.$ $x = 14, -4.$

B. Consecutive Integers

11. Find two consecutive integers whose product is 306.

    Solution. $n(n+1) = 306 \Rightarrow n^2 + n - 306 = 0 \Rightarrow (n+18)(n-17) = 0$. Integers: $17, 18$.

12. The sum of the squares of two consecutive integers is 421.

    Solution. $n^2 + (n+1)^2 = 421 \Rightarrow 2n^2 + 2n + 1 = 421 \Rightarrow n^2 + n - 210 = 0 \Rightarrow (n+15)(n-14) = 0$. Integers: $14, 15$.

13. Find two consecutive even integers whose product is 528.

    Solution. $n(n+2) = 528 \Rightarrow n^2 + 2n - 528 = 0 \Rightarrow (n+24)(n-22) = 0$. Integers: $22, 24$.

14. Find two consecutive odd integers whose product is 399.

    Solution. $n(n+2) = 399 \Rightarrow n^2 + 2n - 399 = 0 \Rightarrow (n+21)(n-19) = 0$. Integers: $19, 21$.

15. The larger of two consecutive integers has a square 61 more than the square of the smaller. Find the integers.

    Solution. $(n+1)^2 - n^2 = 61 \Rightarrow 2n + 1 = 61 \Rightarrow n = 30$. Integers: $30, 31$.

16. Two consecutive even integers have sum 74. Find them.

    Solution. $n + (n+2) = 74 \Rightarrow n = 36$. Integers: $36, 38$.

17. Two consecutive odd integers have sum 116. Find them.

    Solution. $n + (n+2) = 116 \Rightarrow n = 57$. Integers: $57, 59$.

18. The product of three consecutive integers is 990. Find them. (You *do not* need to be able to factor a degree 3 polynomial at this time.)

    Solution. $n(n+1)(n+2) = 990$. Try $9 \cdot 10 \cdot 11 = 990$. Integers: $9, 10, 11$.

C. Two-Number (Sum/Difference/Product)

19. The product of two numbers is 180, and their sum is 27. Find the numbers.

    Solution. $x(27-x)=180 \Rightarrow x^2-27x+180=0 \Rightarrow (x-12)(x-15)=0$. Numbers: $12, 15$.

20. The product of two numbers is 128, and their difference is 8.

    Solution. $x(x-8)=128 \Rightarrow x^2-8x-128=0 \Rightarrow (x-16)(x+8)=0$. Numbers: $16, 8$.

21. The product of two numbers is 168, and their sum is 29.

    Solution. $x(29-x)=168 \Rightarrow x^2-29x+168=0 \Rightarrow (x-21)(x-8)=0$. Numbers: $21, 8$.

22. Two numbers differ by 4, and their product is 320.

    Solution. $x(x-4)=320 \Rightarrow x^2-4x-320=0 \Rightarrow (x-20)(x+16)=0$. Positive pair: 20 and 16.

23. The product of two consecutive even integers is 288. Find them.

    Solution. $n(n+2)=288 \Rightarrow n^2+2n-288=0 \Rightarrow (n+18)(n-16)=0$. Integers: $16, 18$.

24. The product of two consecutive odd integers is 483. Find them.

    Solution. $n(n+2)=483 \Rightarrow n^2+2n-483=0 \Rightarrow (n+23)(n-21)=0$. Integers: $21, 23$.

25. The sum of two numbers is 26, and the sum of their squares is 340. Find the numbers.

    Solution. $x+y=26$, $x^2+y^2=340 \Rightarrow (x+y)^2=340+2xy \Rightarrow 676=340+2xy \Rightarrow xy=168$. Then $t^2-26t+168=0 \Rightarrow (t-12)(t-14)=0$. Numbers: $12, 14$.

26. The sum of two numbers is 31, and their product is 210. Find the numbers.

    Solution. $t^2-31t+210=0 \Rightarrow (t-10)(t-21)=0$. Numbers: $10, 21$.

D. Rectangles (Area and Perimeter)

27. The length of a rectangle is 5 m more than its width. If the area is 204 m^2, find its dimensions.

    Solution. $w(w+5) = 204 \Rightarrow (w+17)(w-12) = 0$. $w = 12$, $l = 17$.

28. The area of a rectangle is 252 cm^2, and the length is 9 cm more than the width.

    Solution. $w(w+9) = 252 \Rightarrow (w+21)(w-12) = 0$. $w = 12$, $l = 21$.

29. A rectangle has area 336 m^2. The length is 4 m greater than twice the width.

    Solution. $w(2w+4) = 336 \Rightarrow 2w^2 + 4w - 336 = 0 \Rightarrow w^2 + 2w - 168 = 0 \Rightarrow (w+14)(w-12) = 0$. $w = 12$, $l = 28$.

30. The length is 3 m less than twice the width. The area is 135 m^2.

    Solution. $w(2w-3) = 135 \Rightarrow 2w^2 - 3w - 135 = 0 \Rightarrow (2w+15)(w-9) = 0$. $w = 9$, $l = 15$.

31. The length is 6 m more than the width. The perimeter is 52 m.

    Solution. $2(l+w) = 52$, $l = w+6 \Rightarrow 2(2w+6) = 52 \Rightarrow w = 10$, $l = 16$.

32. A rectangle has perimeter 70 m. The length is 9 m more than the width.

    Solution. $2(2w+9) = 70 \Rightarrow w = 13$, $l = 22$.

33. The length is 4 m less than three times the width. The area is 532 m^2.

    Solution. $w(3w-4) = 532 \Rightarrow 3w^2 - 4w - 532 = 0 \Rightarrow (w-14)(3w+38) = 0$. $w = 14$, $l = 38$.

34. The area is 168 m^2, and the length is 2 m greater than the width.

    Solution. $w(w+2) = 168 \Rightarrow (w+14)(w-12) = 0$. $w = 12$, $l = 16$.

E. Frames, Borders, and Walkways (Uniform Width)

35. A photo 18 cm × 12 cm is mounted with a uniform border of width x. If the total area is 432 cm^2, find x.

    Solution. $(18+2x)(12+2x) = 432 \Rightarrow 4x^2+60x-216 = 0 \Rightarrow x^2+15x-54 = 0 \Rightarrow (x+18)(x-3) = 0$. $x = 3$ cm.

36. A poster 24 in × 16 in has a frame of width x; the outside dimensions are 30 in × 22 in. Find x.

    Solution. $24 + 2x = 30 \Rightarrow x = 3$ in (and $16 + 2x = 22$ checks).

37. A garden 20 m × 16 m has a walkway of width x so that the walkway area equals 252 m^2. Find x.

    Solution. $(20 + 2x)(16 + 2x) - 320 = 252 \Rightarrow 4x^2 + 72x - 252 = 0 \Rightarrow x^2 + 18x - 63 = 0 \Rightarrow (x + 21)(x - 3) = 0$. $x = 3$ m.

38. A rectangular pool is 12 m × 8 m. A deck of width x surrounds it. If the deck area equals the pool area, find x.

    Solution. $(12+2x)(8+2x)-96 = 96 \Rightarrow 4x^2+40x-96 = 0 \Rightarrow x^2+10x-24 = 0 \Rightarrow (x+12)(x-2) = 0$. $x = 2$ m.

39. A picture 25 cm × 20 cm is framed with uniform width x so that the frame alone has area 306 cm^2. Find x.

    Solution. $(25+2x)(20+2x) - 500 = 306 \Rightarrow 4x^2+90x-306 = 0 \Rightarrow (2x-6)(2x+51) = 0$. $x = 3$ cm.

40. A square patio of side s is surrounded by a uniform border of width x so that the border area equals *three times* the patio area. Express x in terms of s and evaluate for $s = 10$ m.

    Solution. $(s+2x)^2 - s^2 = 3s^2 \Rightarrow 4sx+4x^2 = 3s^2 \Rightarrow 4x^2+4sx-3s^2 = 0 \Rightarrow (2x-s)(2x+3s) = 0$. Positive $x = \frac{s}{2}$; for $s = 10$, $x = 5$ m.

41. A rectangular lawn 18 m × 15 m is bordered by a strip of width x so that the border area is 148 m^2. Find x.

    Solution. $(18 + 2x)(15 + 2x) - 270 = 148 \Rightarrow 4x^2 + 66x - 148 = 0 \Rightarrow 2x^2 + 33x - 74 = 0 \Rightarrow (2x + 37)(x - 2) = 0$. $x = 2$ m.

42. A rug 12 ft × 9 ft sits centered in a room; the uncovered floor (uniform margin) has area 162 ft^2. Find the margin width x.

    Solution. $(12 + 2x)(9 + 2x) - 108 = 162 \Rightarrow 4x^2 + 42x - 162 = 0 \Rightarrow 2x^2 + 21x - 81 = 0 \Rightarrow (2x + 27)(x - 3) = 0$. $x = 3$ ft.

F. Area Change by Adding/Removing Strips

43. A square garden's side is increased by 3 m, and the area increases by 129 m^2. Find the original side.

    Solution. $(x+3)^2 - x^2 = 129 \Rightarrow 6x + 9 = 129 \Rightarrow x = 20$ m.

44. A square's side is decreased by 5 cm, and the area decreases by 95 cm^2. Find the original side.

    Solution. $x^2 - (x-5)^2 = 95 \Rightarrow 10x - 25 = 95 \Rightarrow x = 12$ cm.

45. A rectangular field 24 m $\times$ 10 m is reduced by fencing off a strip of width x along one long side and one short side. The remaining area is 176 m^2. Find x.

    Solution. $(24-x)(10-x) = 176 \Rightarrow x^2 - 34x + 64 = 0 \Rightarrow (x-2)(x-32) = 0$. Feasible $x = 2$ m.

46. A rectangle 30 m $\times$ 20 m is enlarged by adding x to *each dimension* so that the new area is 936 m^2. Find x.

    Solution. $(30+x)(20+x) = 936 \Rightarrow x^2 + 50x - 336 = 0 \Rightarrow (x+56)(x-6) = 0$. $x = 6$ m.

G. Triangles (Base–Height Relationships)

47. The base of a triangle is 3 cm more than twice its height. If the area is 76 cm^2, find base and height.

    Solution. $\frac{1}{2}(2h+3)h = 76 \Rightarrow 2h^2 + 3h - 152 = 0 \Rightarrow (2h+19)(h-8) = 0$. $h = 8$, base $= 19$.

48. The base exceeds the height by 7 cm, and the area is 99 cm^2.

    Solution. $\frac{1}{2}h(h+7) = 99 \Rightarrow h^2 + 7h - 198 = 0 \Rightarrow (h+18)(h-11) = 0$. $h = 11$, base $= 18$.

49. The base is 5 cm less than three times the height. The area is 56 cm^2.

    Solution. $\frac{1}{2}(3h-5)h = 56 \Rightarrow 3h^2 - 5h - 112 = 0 \Rightarrow (3h+16)(h-7) = 0$. $h = 7$, base $= 16$.

50. The height is 4 cm more than the base; the area is 96 cm^2.

    Solution. $\frac{1}{2}b(b+4) = 96 \Rightarrow b^2 + 4b - 192 = 0 \Rightarrow (b+16)(b-12) = 0$. $b = 12$, $h = 16$.

H. Numbers with Reciprocals (Factorable)

51. The sum of a number and its reciprocal is $\dfrac{13}{6}$. Find the number(s).

    Solution. $x + \dfrac{1}{x} = \dfrac{13}{6} \Rightarrow 6x^2 - 13x + 6 = 0 \Rightarrow (3x-2)(2x-3) = 0$. $x = \frac{2}{3}, \frac{3}{2}$.

52. A number equals its reciprocal. Find the number(s).

    Solution. $x = \dfrac{1}{x} \Rightarrow x^2 - 1 = 0 \Rightarrow (x-1)(x+1) = 0$. $x = \pm 1$.

53. The sum of a number and its reciprocal is $\dfrac{5}{2}$. Find the number(s).

    Solution. $x + \dfrac{1}{x} = \dfrac{5}{2} \Rightarrow 2x^2 - 5x + 2 = 0 \Rightarrow (2x-1)(x-2) = 0$. $x = \frac{1}{2}$, 2.

I. Mixed Quick Practice (All Factorable)

54. The square of a number is 48 more than twice the number.

    Solution. $x^2 = 2x + 48 \Rightarrow x^2 - 2x - 48 = 0 \Rightarrow (x-8)(x+6) = 0$. $x = 8, -6$.

55. A number times 7 more than itself is 120.

    Solution. $x(x+7) = 120 \Rightarrow x^2 + 7x - 120 = 0 \Rightarrow (x+15)(x-8) = 0$. $x = 8, -15$.

56. The square of a number is 100 less than 25 times the number.

    Solution. $x^2 = 25x - 100 \Rightarrow x^2 - 25x + 100 = 0 \Rightarrow (x-5)(x-20) = 0$. $x = 5, 20$.

57. The product of two consecutive integers is 380.

    Solution. $n(n+1) = 380 \Rightarrow n^2 + n - 380 = 0 \Rightarrow (n+20)(n-19) = 0$. Integers: $19, 20$.

58. The product of two consecutive even integers is 960.

    Solution. $n(n+2) = 960 \Rightarrow n^2 + 2n - 960 = 0 \Rightarrow (n+32)(n-30) = 0$. Integers: $30, 32$.

59. The product of two consecutive odd integers is 783.

    Solution. $n(n+2) = 783 \Rightarrow n^2 + 2n - 783 = 0 \Rightarrow (n+29)(n-27) = 0$. Integers: $27, 29$.

60. The area of a rectangle is 308 m^2. The length is 8 m more than the width.

    Solution. $w(w+8) = 308 \Rightarrow w^2 + 8w - 308 = 0 \Rightarrow (w+22)(w-14) = 0$. $w = 14$, $l = 22$.

61. A park 40 m $\times$ 24 m is expanded by a uniform strip x so the new area is 1380 m^2. Find x.

    Solution. $(40+2x)(24+2x) = 1380 \Rightarrow 4x^2+128x-420 = 0 \Rightarrow x^2+32x-105 = 0 \Rightarrow (x+35)(x-3) = 0$. $x = 3$ m.

62. A kiosk 12 ft $\times$ 10 ft is surrounded by a walkway of width x so that the walkway area equals 168 ft^2. Find x.

    Solution. $(12+2x)(10+2x) - 120 = 168 \Rightarrow 4x^2 + 44x - 168 = 0 \Rightarrow 2x^2 + 22x - 84 = 0 \Rightarrow 2(x+14)(x-3) = 0$. $x = 3$ ft.

63. Two positive numbers differ by 5, and their product is 234. Find the numbers.

    Solution. $x(x-5) = 234 \Rightarrow x^2 - 5x - 234 = 0 \Rightarrow (x-18)(x+13) = 0$. Positive pair: 18 and 13.