Extra Practice

Math 10/20 · Mr. Merrick · January 20, 2026

1. Find the equations of the two lines that are 4 units from the line $5x + 12y = 8$.

[Lines that stay the same distance apart must be parallel (same slope). So we look for all lines parallel to $5x + 12y = 8$ that are 4 units away.

If we only change the constant on the right, the slope stays the same:

$$5x + 12y = k.$$

For lines of the form $Ax + By = k$, the distance between $Ax + By = k_1$ and $Ax + By = k_2$ is

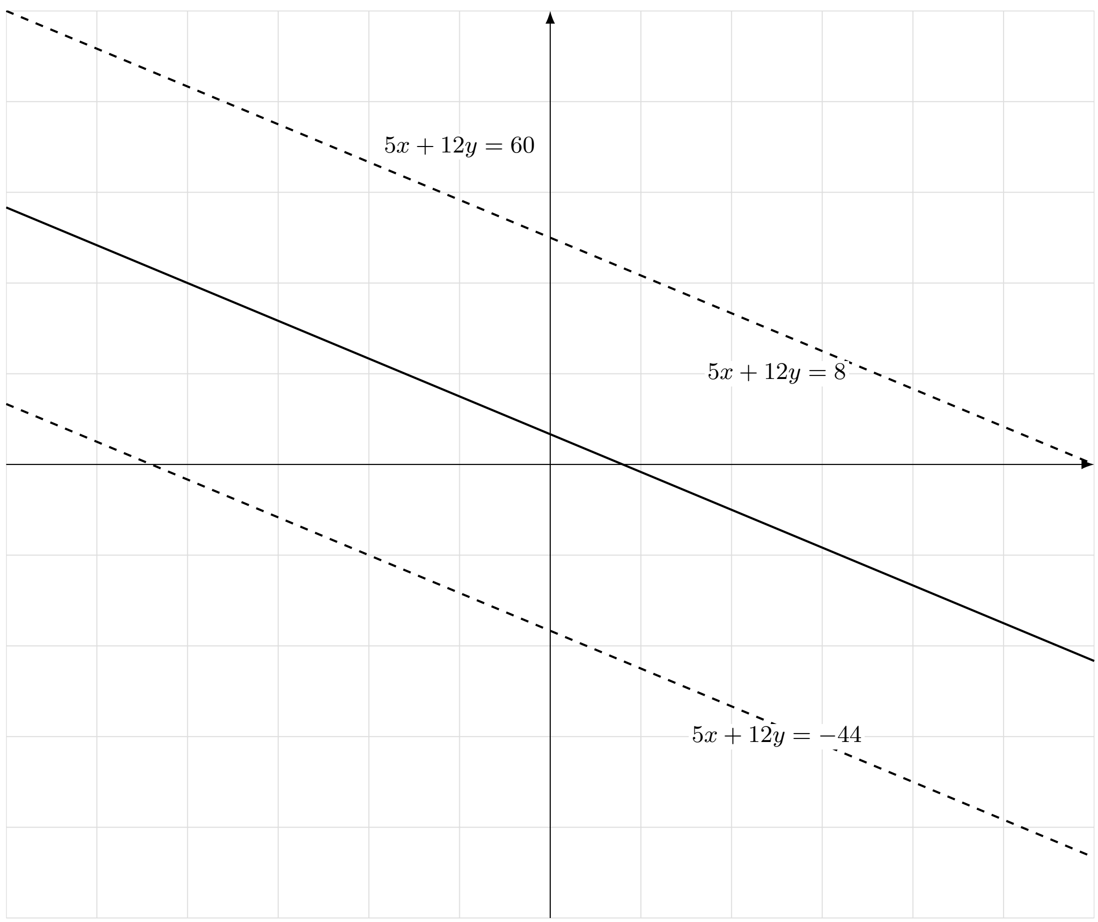
$$\frac{|k_2 - k_1|}{\sqrt{A^2 + B^2}}.$$

Here $A = 5$, $B = 12$, so $\sqrt{5^2 + 12^2} = 13$.

Set the distance equal to 4:

$$\frac{|k - 8|}{13} = 4 \Rightarrow |k - 8| = 52 \Rightarrow k = 60 \text{ or } k = -44.$$

So the two lines are $\boxed{5x + 12y = 60 \text{ and } 5x + 12y = -44}$.]



2. A line intersects the positive x- and y-axes and contains the point $P(-2.5, 3)$. One of its intercepts is 4. Find the slope of the line.

[The line crosses the x-axis at some positive point $(a, 0)$ and the y-axis at some positive point $(0, b)$. One intercept is 4, so either $a = 4$ or $b = 4$.

If a line goes through $(a, 0)$ and $(0, b)$, then it can be written as

$$\frac{x}{a} + \frac{y}{b} = 1,$$

because it is true for $(a, 0)$ and also true for $(0, b)$.

Try $a = 4$:

$$\frac{x}{4} + \frac{y}{b} = 1.$$

Plug in $P(-2.5, 3)$:

$$\frac{-2.5}{4} + \frac{3}{b} = 1 \Rightarrow -\frac{5}{8} + \frac{3}{b} = 1 \Rightarrow \frac{3}{b} = \frac{13}{8} \Rightarrow b = \frac{24}{13}.$$

Now use the two intercept points $(4, 0)$ and $\left(0, \frac{24}{13}\right)$ to get slope:

$$m = \frac{\frac{24}{13} - 0}{0 - 4} = -\frac{6}{13}.$$

Check the other option $b = 4$:

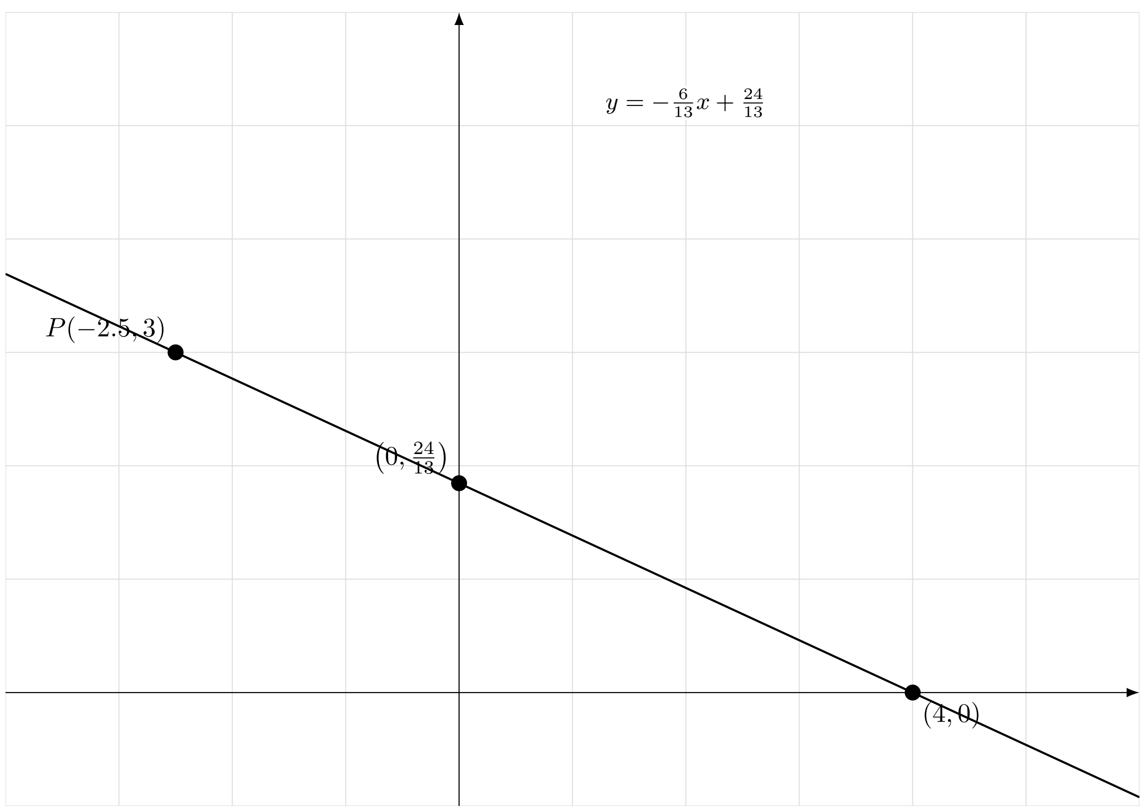
$$\frac{x}{a} + \frac{y}{4} = 1.$$

Plug in $P(-2.5, 3)$:

$$\frac{-2.5}{a} + \frac{3}{4} = 1 \Rightarrow \frac{-2.5}{a} = \frac{1}{4} \Rightarrow a = -10,$$

but that is not allowed because the intercept must be on the positive x-axis.

So the slope is $\boxed{-\frac{6}{13}}$.]



3. Let b be a real number. The two lines whose equations are $2x - y = 5b$ and $4x + y = 6b^2 - 17b$ intersect at a point P. Determine all values of b so that P lies below the line $x - y = 10$.

[First find the intersection point $P(x, y)$ in terms of b.

Add the equations so y cancels:

$$(2x - y) + (4x + y) = 5b + (6b^2 - 17b) \Rightarrow 6x = 6b^2 - 12b \Rightarrow x = b^2 - 2b.$$

Substitute into $2x - y = 5b$:

$$2(b^2 - 2b) - y = 5b \Rightarrow y = 2b^2 - 9b.$$

So

$$P\big(b^2 - 2b,\ 2b^2 - 9b\big).$$

Rewrite $x - y = 10$ as $y = x - 10$. Being below the line means $y < x - 10$.

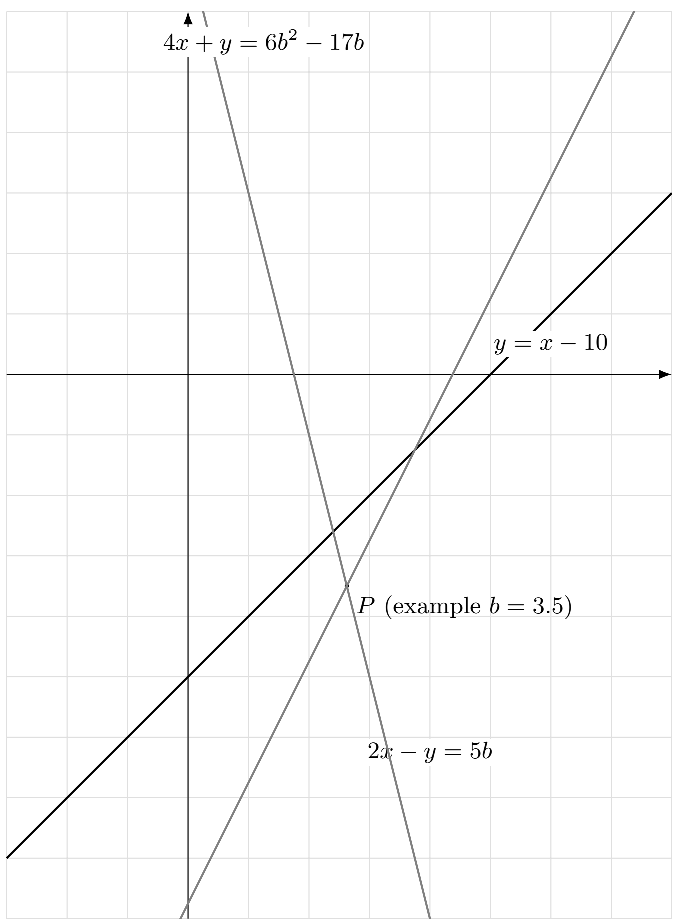
Substitute x and y from P:

$$2b^2 - 9b < (b^2 - 2b) - 10 \Rightarrow b^2 - 7b + 10 < 0 \Rightarrow (b - 2)(b - 5) < 0.$$

This is negative between the roots, so

$$2 < b < 5.$$

Thus $\boxed{2 < b < 5}$.]



4. The vertices of $\triangle ABC$ are $A(0,0)$, $B(8,6)$, and $C(3,5)$. Find the equation of the vertical line that divides the triangle into two regions of equal area.

[First find the total area. With $A(0,0)$, we can use

$$\text{Area} = \frac{1}{2}|x_1y_2 - y_1x_2|$$

with $(x_1, y_1) = (8,6)$ and $(x_2, y_2) = (3,5)$:

$$\text{Area} = \frac{1}{2}|8 \cdot 5 - 6 \cdot 3| = \frac{1}{2}|40 - 18| = 11.$$

Half is $\frac{11}{2}$.

Let the vertical line be $x = k$.

Find the lines of the triangle:

$$AB: \ y = \frac{3}{4}x, \qquad AC: \ y = \frac{5}{3}x, \qquad BC: \ y = \frac{1}{5}x + \frac{22}{5}.$$

If $0 \le k \le 3$, the left piece is a triangle between AB and AC. At $x = k$ the vertical length is

$$\frac{5}{3}k - \frac{3}{4}k = \frac{11}{12}k.$$

So left area is

$$A_L = \frac{1}{2}(k)\left(\frac{11}{12}k\right) = \frac{11}{24}k^2.$$

Setting $\frac{11}{24}k^2 = \frac{11}{2}$ gives $k^2 = 12$, so $k \approx 3.46$, not in $[0,3]$.

So $3 \le k \le 8$. It is easier to find the area on the right and subtract from 11.

At $x = k$:

$$y_{AB} = \frac{3}{4}k, \qquad y_{BC} = \frac{1}{5}k + \frac{22}{5}.$$

The vertical length is

$$y_{BC} - y_{AB} = \frac{88 - 11k}{20} = \frac{11(8-k)}{20}.$$

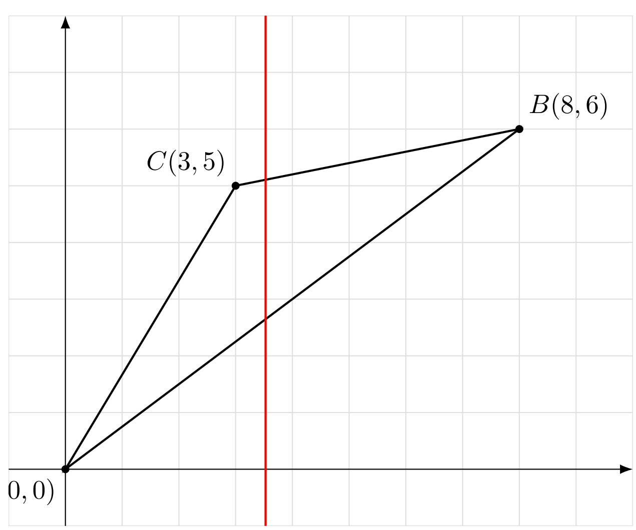
The horizontal distance to $B(8,6)$ is $(8-k)$. So right area is

$$A_R = \frac{1}{2}\left(\frac{11(8-k)}{20}\right)(8-k) = \frac{11}{40}(8-k)^2.$$

We want $11 - A_R = \frac{11}{2}$:

$$11 - \frac{11}{40}(8-k)^2 = \frac{11}{2} \Rightarrow (8-k)^2 = 20 \Rightarrow 8 - k = 2\sqrt{5} \Rightarrow k = 8 - 2\sqrt{5}.$$

So the line is $\boxed{x = 8 - 2\sqrt{5}}$.]



5. Two of the vertices of $\triangle ABC$ are $A(-2, 6)$ and $B(4, -2)$. The third vertex C lies on the line $2x - 3y = 6$. Find the coordinates of C if the area of $\triangle ABC$ is 32.

[Let $C = (x, y)$ and remember C must satisfy $2x - 3y = 6$.

Use the coordinate area formula:

$$\text{Area} = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|.$$

With $A(-2, 6)$, $B(4, -2)$, and $C(x, y)$:

$$32 = \frac{1}{2}\left|(-2)(-2 - y) + 4(y - 6) + x(6 - (-2))\right|.$$

Simplify inside:

$$(-2)(-2 - y) = 4 + 2y, \quad 4(y - 6) = 4y - 24, \quad x(8) = 8x.$$

So

$$32 = \frac{1}{2}|8x + 6y - 20| \Rightarrow |8x + 6y - 20| = 64.$$

That gives two equations:

$$8x + 6y = 84 \quad \text{or} \quad 8x + 6y = -44.$$

Now solve each with $2x - 3y = 6$.

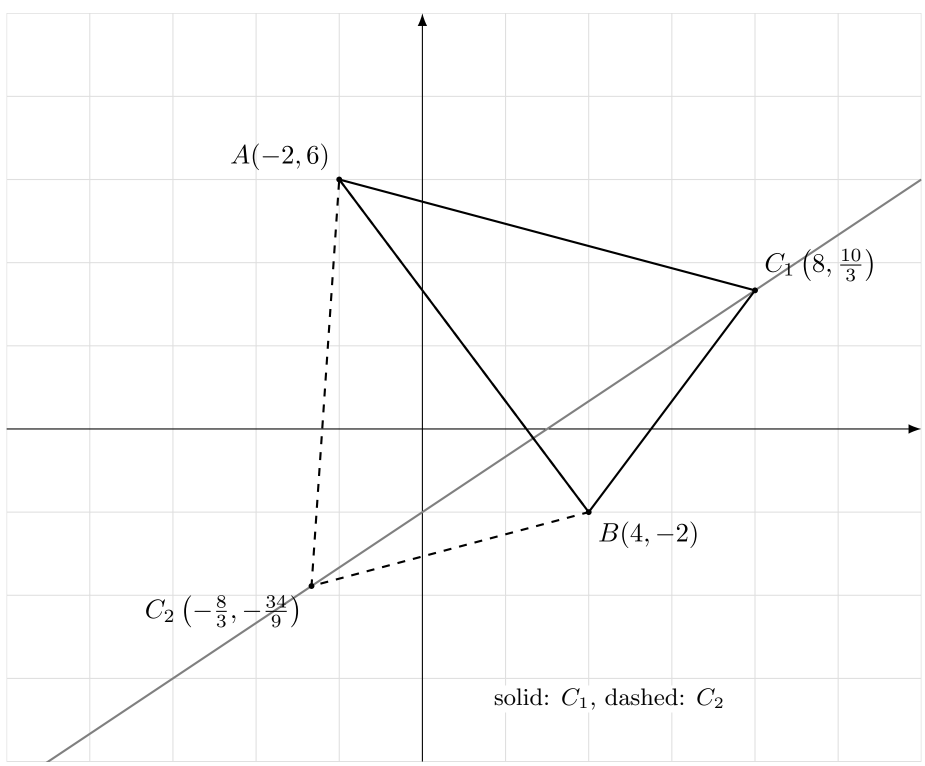
Case 1:

$$\begin{cases} 8x + 6y = 84 \\ 2x - 3y = 6 \end{cases} \Rightarrow C = \left(8, \frac{10}{3}\right).$$

Case 2:

$$\begin{cases} 8x + 6y = -44 \\ 2x - 3y = 6 \end{cases} \Rightarrow C = \left(-\frac{8}{3}, -\frac{34}{9}\right).$$

So $\boxed{C = \left(8, \frac{10}{3}\right) \text{ or } C = \left(-\frac{8}{3}, -\frac{34}{9}\right)}$.]



6. Line L_1 has x-intercept at $A(8, 0)$. Line L_2 is perpendicular to L_1 and has y-intercept at $B(0, 6)$. The two lines intersect at a point C on the line $y = x$. Find the equations of the lines L_1 and L_2.

[Let the slope of L_1 be m. Since L_1 goes through $(8, 0)$:

$$L_1 : \; y = m(x - 8).$$

A line perpendicular to slope m has slope $-\frac{1}{m}$, so

$$L_2 : \; y = -\frac{1}{m}x + 6$$

because its y-intercept is 6.

The intersection point C is on $y = x$, so at C we can replace y with x.

From L_2:

$$x = -\frac{1}{m}x + 6 \Rightarrow x = \frac{6m}{m + 1}.$$

From L_1:

$$x = m(x - 8) \Rightarrow x = \frac{8m}{m - 1}.$$

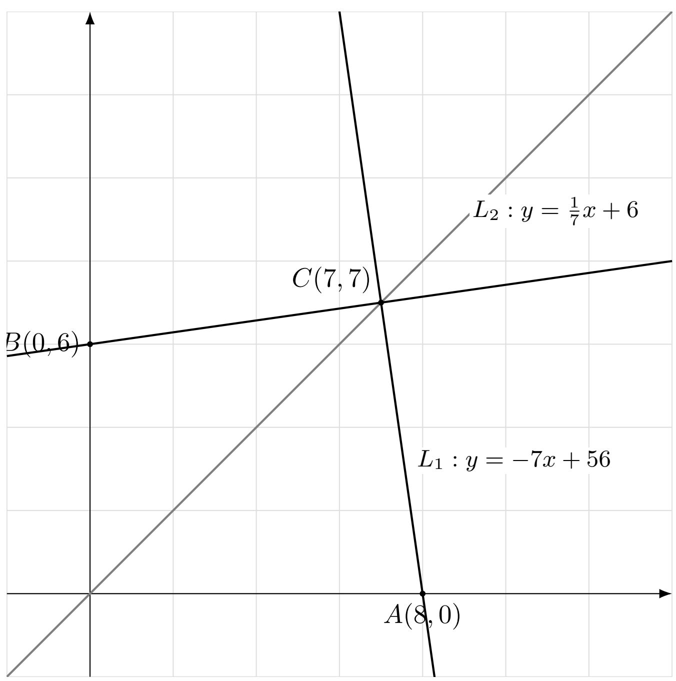
Set them equal:

$$\frac{6m}{m + 1} = \frac{8m}{m - 1} \Rightarrow \frac{6}{m + 1} = \frac{8}{m - 1} \Rightarrow 6(m - 1) = 8(m + 1) \Rightarrow m = -7.$$

So

$$L_1 : \; y = -7(x - 8) = -7x + 56, \qquad L_2 : \; y = \frac{1}{7}x + 6.$$

Thus $\boxed{L_1 : y = -7x + 56 \text{ and } L_2 : y = \frac{1}{7}x + 6}$.]



7. The triangle ABC has vertices $A(3,2)$, $B(8,2)$, and $C(5,6)$.

   (a) Calculate the area of this triangle.

   [A and B have the same y-value, so AB is horizontal.

   $$|AB| = 8 - 3 = 5, \qquad \text{height} = 6 - 2 = 4.$$

   $$\text{Area} = \frac{1}{2}(5)(4) = 10$$

   ]

   (b) Calculate the length of the altitude AD.

   [Find the equation of BC.

   Slope:

   $$m_{BC} = \frac{6-2}{5-8} = -\frac{4}{3}.$$

   Using $B(8,2)$:

   $$y - 2 = -\frac{4}{3}(x-8) \Rightarrow 4x + 3y - 38 = 0.$$

   Distance from $A(3,2)$ to the line $4x + 3y - 38 = 0$:

   $$d = \frac{|4(3) + 3(2) - 38|}{\sqrt{4^2+3^2}} = \frac{|12+6-38|}{5} = \frac{20}{5} = 4.$$

   So $\boxed{AD = 4}$.]

   (c) A line parallel to AD intersects AB at M and BC at N. If $MN = 3$, find the coordinates of the points M and N.

   [Since BC has slope $-\frac{4}{3}$, a perpendicular line (like AD) has slope $\frac{3}{4}$, so a direction vector is $(4,3)$. That direction has length $\sqrt{4^2+3^2} = 5$.

   To make length 3, scale by $\frac{3}{5}$:

   $$\overrightarrow{MN} = \left(\frac{12}{5}, \frac{9}{5}\right).$$

   Let $M = (t, 2)$ on AB (because AB is $y = 2$). Then

   $$N = \left(t + \frac{12}{5}, \frac{19}{5}\right).$$

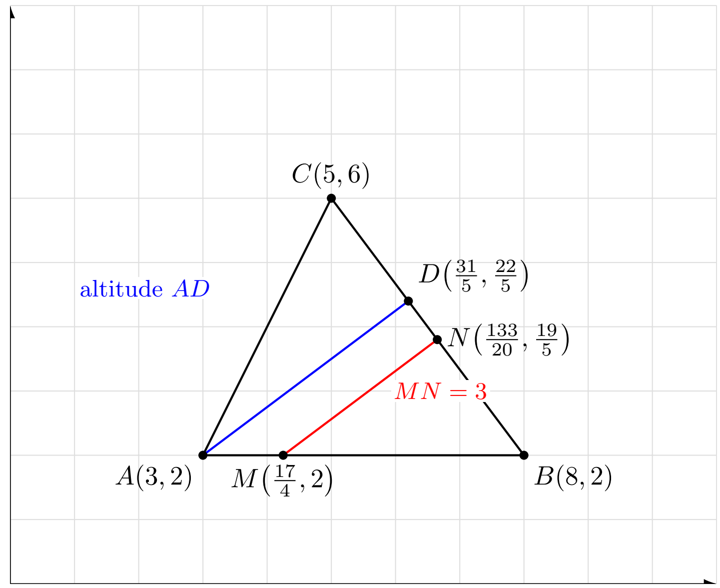
   Because N lies on BC, use $4x + 3y = 38$:

   $$4\left(t + \frac{12}{5}\right) + 3\left(\frac{19}{5}\right) = 38 \Rightarrow 4t + 21 = 38 \Rightarrow t = \frac{17}{4}.$$

   So

   $$M = \left(\frac{17}{4}, 2\right), \quad N = \left(\frac{17}{4} + \frac{12}{5}, \frac{19}{5}\right) = \left(\frac{133}{20}, \frac{19}{5}\right).$$

   Thus $\boxed{M\left(\frac{17}{4}, 2\right),\ N\left(\frac{133}{20}, \frac{19}{5}\right)}$.]



$C(5,6)$
$D\left(\frac{31}{5}, \frac{22}{5}\right)$
altitude AD
$N\left(\frac{133}{20}, \frac{19}{5}\right)$
$MN = 3$
$A(3,2)$
$M\left(\frac{17}{4}, 2\right)$
$B(8,2)$