More Practice

Math 10 · Mr. Merrick · February 3, 2026

1. Solve the system (4 equations, 4 variables):

$$\begin{aligned} x_1 + 2x_2 - x_3 &= 2 \\ 2x_1 + 5x_2 + x_4 &= 16 \\ -x_2 + 3x_3 + 2x_4 &= 15 \\ 3x_1 + 4x_2 + x_3 - x_4 &= 10 \end{aligned}$$

2. Solve the system:

$$\begin{aligned} 2x_1 - x_2 + x_3 + 3x_4 &= 4 \\ x_1 + 2x_3 - x_4 &= -5 \\ 3x_1 + x_2 - x_3 + 2x_4 &= 1 \\ 2x_2 + x_3 + x_4 &= 5 \end{aligned}$$

3. Solve the system. If there is no solution or infinitely many solutions, state which and explain using row-reduction ideas.

$$\begin{aligned} x_1 - 2x_2 + x_3 + x_4 &= 1 \\ 2x_1 - 4x_2 + 2x_3 + 2x_4 &= 5 \\ x_1 + x_2 - x_3 &= 0 \\ x_2 + 2x_3 - x_4 &= 3 \end{aligned}$$

4. The reduced row-echelon form of an augmented matrix for variables x_1, x_2, x_3, x_4 is:

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 5 \\ 0 & 1 & -1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Describe the solution set.

5. Determine the value(s) of k for which the system has a unique solution, infinitely many solutions, or no solution:

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 4 \\ 2x_1 + 2x_2 + 2x_3 + kx_4 &= 8 \\ x_1 - x_2 + 2x_3 - x_4 &= 1 \\ 3x_1 + x_2 - x_3 + 2x_4 &= 7 \end{aligned}$$

6. Consider the system

$$\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned}$$

where $ad - bc \neq 0$.

Prove that the solution to this system is

$$x = \frac{ed - bf}{ad - bc} \qquad \text{and} \qquad y = \frac{af - ec}{ad - bc}.$$

7. Determine the value(s) of k for which the system has infinitely many solutions or no solution:

$$\begin{aligned} x_1 + 2x_2 - x_3 + x_4 &= 3 \\ 3x_1 + 6x_2 - 3x_3 + 3x_4 &= k \\ x_1 - x_2 + x_3 &= 1 \\ x_2 + x_3 - x_4 &= 0 \end{aligned}$$

8. Consider the system

$$\begin{aligned} x + y + z &= 3 \\ 2x + 2y + 2z &= 6 \\ x - y + z &= 1 \end{aligned}$$

   (a) Describe geometrically what each equation represents in $\mathbb{R}^3$.

   (b) Explain why the first two equations represent the same plane.

   (c) Prove that the system has infinitely many solutions by describing the geometric intersection.

9. Consider the system

$$\begin{aligned} x + y + z &= 4 \\ 2x + 2y + 2z &= 8 \\ x + y + z &= 1 \end{aligned}$$

   (a) Describe geometrically what each equation represents in $\mathbb{R}^3$.

   (b) Explain the relationship between the first and third equations.

   (c) Prove that the system has no solution by describing the geometric situation.

10. Consider the system

$$\begin{aligned} x + y + z &= 6 \\ x - y + z &= 2 \\ 2x + y - z &= 5 \end{aligned}$$

   (a) Describe geometrically what each equation represents in $\mathbb{R}^3$.

   (b) Explain why no two of the planes are the same or parallel.

   (c) Prove that the system has a unique solution by describing the geometric intersection.