More Practice

Math 10 · Mr. Merrick · February 3, 2026

1. Solve the system (4 equations, 4 variables):

$$\begin{aligned} x_1 + 2x_2 - x_3 &= 2 \\ 2x_1 + 5x_2 + x_4 &= 16 \\ -x_2 + 3x_3 + 2x_4 &= 15 \\ 3x_1 + 4x_2 + x_3 - x_4 &= 10 \end{aligned}$$

[$(x_1, x_2, x_3, x_4) = (1, 2, 3, 4)$.]

2. Solve the system:

$$\begin{aligned} 2x_1 - x_2 + x_3 + 3x_4 &= 4 \\ x_1 + 2x_3 - x_4 &= -5 \\ 3x_1 + x_2 - x_3 + 2x_4 &= 1 \\ 2x_2 + x_3 + x_4 &= 5 \end{aligned}$$

[$(x_1, x_2, x_3, x_4) = (-2, 1, 0, 3)$.]

3. Solve the system. If there is no solution or infinitely many solutions, state which and explain using row-reduction ideas.

$$\begin{aligned} x_1 - 2x_2 + x_3 + x_4 &= 1 \\ 2x_1 - 4x_2 + 2x_3 + 2x_4 &= 5 \\ x_1 + x_2 - x_3 &= 0 \\ x_2 + 2x_3 - x_4 &= 3 \end{aligned}$$

[No solution, because equation 2 has the same left side as 2·(equation 1) but the constant does not match ($5 \neq 2 \cdot 1$). This creates a contradiction.]

4. The reduced row-echelon form of an augmented matrix for variables x_1, x_2, x_3, x_4 is:

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 5 \\ 0 & 1 & -1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Describe the solution set. [Let $x_3 = t$ (free variable). Then $x_1 + 2x_3 = 5 \Rightarrow x_1 = 5 - 2t$, and $x_2 - x_3 = -2 \Rightarrow x_2 = -2 + t$, and $x_4 = 3$.
So $(x_1, x_2, x_3, x_4) = (5 - 2t,\ -2 + t,\ t,\ 3)$.]

5. Determine the value(s) of k for which the system has a unique solution, infinitely many solutions, or no solution:

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 4 \\ 2x_1 + 2x_2 + 2x_3 + kx_4 &= 8 \\ x_1 - x_2 + 2x_3 - x_4 &= 1 \\ 3x_1 + x_2 - x_3 + 2x_4 &= 7 \end{aligned}$$

[Subtract 2·(equation 1) from equation 2 to get $(k - 2)x_4 = 0$.
If $k \neq 2$, then $x_4 = 0$ and the remaining equations determine a unique solution:

$$(x_1, x_2, x_3, x_4) = \left(\tfrac{19}{10}, \tfrac{17}{10}, \tfrac{2}{5}, 0\right).$$

If $k = 2$, equation 2 becomes redundant, giving one free variable and infinitely many solutions.
No solution never occurs for this system.]

6. Consider the system

$$ax + by = e$$
$$cx + dy = f$$

where $ad - bc \neq 0$.

Prove that the solution to this system is

$$x = \frac{ed - bf}{ad - bc} \qquad \text{and} \qquad y = \frac{af - ec}{ad - bc}.$$

[Multiply the first equation by d and the second by b:

$$adx + bdy = ed, \qquad bcx + bdy = bf.$$

Subtracting gives

$$(ad - bc)x = ed - bf.$$

Since $ad - bc \neq 0$, divide to obtain $x = \dfrac{ed - bf}{ad - bc}$.

Similarly, multiply the first equation by c and the second by a:

$$acx + bcy = ec, \qquad acx + ady = af.$$

Subtracting gives

$$(ad - bc)y = af - ec,$$

so $y = \dfrac{af - ec}{ad - bc}$.]

7. Determine the value(s) of k for which the system has infinitely many solutions or no solution:

$$\begin{aligned} x_1 + 2x_2 - x_3 + x_4 &= 3 \\ 3x_1 + 6x_2 - 3x_3 + 3x_4 &= k \\ x_1 - x_2 + x_3 &= 1 \\ x_2 + x_3 - x_4 &= 0 \end{aligned}$$

[Equation 2 has the same left side as 3·(equation 1).
If $k = 9$, equation 2 is redundant and the system has infinitely many solutions.
If $k \neq 9$, the system has no solution.]

8. Consider the system

$$\begin{aligned} x+y+z&=3\\ 2x+2y+2z&=6\\ x-y+z&=1 \end{aligned}$$

(a) Describe geometrically what each equation represents in $\mathbb{R}^3$.

(b) Explain why the first two equations represent the same plane.

(c) Prove that the system has infinitely many solutions by describing the geometric intersection.

[(a) Each equation represents a plane in three-dimensional space.
(b) The second equation is exactly 2 times the first equation, so both equations describe the same plane.
(c) The system therefore consists of two distinct planes: the plane $x + y + z = 3$ and the plane $x - y + z = 1$. Since these planes are not parallel, they intersect in a line. Every point on this line satisfies all three equations, so the system has infinitely many solutions.]

9. Consider the system

$$\begin{aligned} x+y+z&=4\\ 2x+2y+2z&=8\\ x+y+z&=1 \end{aligned}$$

(a) Describe geometrically what each equation represents in $\mathbb{R}^3$.

(b) Explain the relationship between the first and third equations.

(c) Prove that the system has no solution by describing the geometric situation.

[(a) Each equation represents a plane in three-dimensional space.
(b) The first and third equations have identical left-hand sides but different constants, so they represent parallel planes.
(c) Parallel planes do not intersect, so there is no point that satisfies both $x + y + z = 4$ and $x + y + z = 1$. Therefore, no point satisfies all three equations and the system has no solution.]

10. Consider the system

$$\begin{aligned} x+y+z&=6\\ x-y+z&=2\\ 2x+y-z&=5 \end{aligned}$$

(a) Describe geometrically what each equation represents in $\mathbb{R}^3$.

(b) Explain why no two of the planes are the same or parallel.

(c) Prove that the system has a unique solution by describing the geometric intersection.

[(a) Each equation represents a plane in three-dimensional space.
(b) None of the equations is a scalar multiple of another, so the planes are distinct and not parallel.
(c) The first two planes intersect in a line. The third plane is not parallel to this line and does not contain it, so it intersects the line at exactly one point. Therefore, the three planes intersect at a single point, and the system has a unique solution corresponding to that point of intersection.]