More Practice with Matrices

Math 10 · Mr. Merrick · February 2, 2026

1. Turn the following system into an augmented matrix:

$$2x - 3y = 7$$
$$-x + 4y = -5$$

$\left[\left[\begin{array}{cc|c} 2 & -3 & 7 \\ -1 & 4 & -5 \end{array}\right] \right]$

2. Start with

$$\begin{bmatrix} 1 & -2 & 5 \\ 3 & 0 & -6 \\ -1 & 4 & 2 \end{bmatrix}$$

Perform each operation one after another, writing the full matrix after each step:

(a) $R_2 \leftarrow R_2 - 3R_1$

(b) $R_3 \leftarrow R_3 + R_1$

(c) swap R_2 and R_3

[(a) $\begin{bmatrix} 1 & -2 & 5 \\ 0 & 6 & -21 \\ -1 & 4 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 & 5 \\ 0 & 6 & -21 \\ 0 & 2 & 7 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & -2 & 5 \\ 0 & 2 & 7 \\ 0 & 6 & -21 \end{bmatrix}$]

3. Solve the system using row reduction:

$$x + y = 5$$
$$2x - y = 4$$

$\left[\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 2 & -1 & 4 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & -3 & -6 \end{array}\right] \to y = 2,\ x = 3 \right]$

4. Solve the system:

$$\begin{aligned} x + y + z &= 6 \\ 2x - y + z &= 3 \\ -x + 2y + z &= 5 \end{aligned}$$

[The system has a unique solution. From (2)−2·(1) we get $-3y - z = -9$, and from (3)+(1) we get $3y + 2z = 11$. Solving gives $y = \frac{7}{3}$ and $z = 2$. Substituting into (1) gives $x = \frac{5}{3}$. Thus $(x, y, z) = \left(\frac{5}{3},\ \frac{7}{3},\ 2\right)$.]

5. The reduced row-echelon form of an augmented matrix is:

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Describe the solution set. [Let $z = t$. Then $x + 2z = 4 \Rightarrow x = 4 - 2t$, and $y - z = 3 \Rightarrow y = 3 + t$. The system has infinitely many solutions: $(x, y, z) = (4 - 2t,\ 3 + t,\ t)$.]

6. A row-reduction ends with:

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 5 \end{array}\right]$$

What does this tell you about the system? [The last row represents $0 = 5$, which is impossible, so the system has no solution.]

7. Solve the equation and describe the solution set:

$$3x - 6y = 12$$

[One equation in two variables gives infinitely many solutions. Simplifying: $x - 2y = 4 \Rightarrow x = 4 + 2y$.]

8. Determine the value(s) of k for which the system has a unique solution, no solution, or infinitely many solutions:

$$x + 2y = 6$$
$$2x + ky = 12$$

[The augmented matrix reduces to $\left[\begin{array}{cc|c} 1 & 2 & 6 \\ 0 & k-4 & 0 \end{array}\right]$.
If $k \neq 4$, there is a pivot in the y-column and the system has a unique solution $(6, 0)$.
If $k = 4$, the second row becomes $0 = 0$, giving infinitely many solutions.
There is no value of k that produces no solution.]

9. Determine the value(s) of k for which the system has no solution:

$$2x - 3y = 1$$
$$4x - 6y = k$$

[The left side of the second equation is twice the left side of the first.
If $k = 2$, the equations represent the same line and there are infinitely many solutions.
If $k \neq 2$, the equations represent parallel lines and there is no solution.]

10. Determine the value(s) of k for which the system has a unique solution, no solution, or infinitely many solutions:

$$\begin{aligned} x + y + z &= 3 \\ 2x + 2y + 2z &= k \\ x - y + z &= 1 \end{aligned}$$

[The second equation has the same left side as twice the first.
If $k = 6$, the system is consistent and has infinitely many solutions.
If $k \neq 6$, the system is inconsistent and has no solution.
There is no value of k that gives a unique solution.]

11. A cinema sells adult tickets for \$12 and student tickets for \$8. A total of 50 tickets were sold for \$480.

    (a) Write a system of equations.

    (b) Solve the system.

[(a) $a + s = 50$ and $12a + 8s = 480$.
(b) Solving gives $s = 30$ and $a = 20$.]

12. For the augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & 4 & -2 & k \\ 0 & 1 & 1 & 2 \end{array}\right]$$

determine the value(s) of k for which the system has infinitely many solutions, and the value(s) for which it has no solution. [Row-reducing gives a row of the form $[0\ 0\ 0 \mid k - 6]$.
If $k = 6$, the system has infinitely many solutions.
If $k \neq 6$, the system has no solution.]