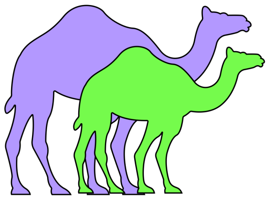
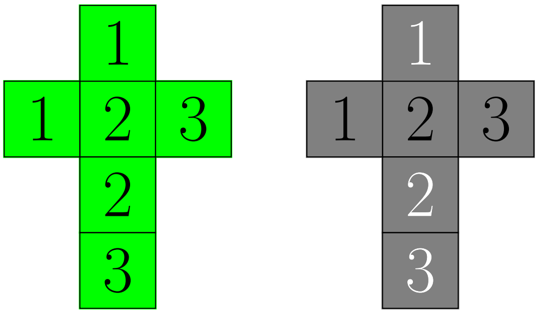


# Racing Camel Probability

In a normal game of 'Camel-Up', there are 6 *regular* dice placed in the pyramid: red, yellow, blue, green, and purple. There is one grey *crazy* dice (this controls both the white and black camels).




*Regular* Green Dice     *Crazy* Dice

1. How many ways can 5 dice be rolled from the pyramid of 6? Does the order of the dice rolled matter in the game?

2. What is the probability that the yellow die is the first rolled?

3. What is the probability that the black die is the first rolled?

4. What is the probability that the blue die is the first rolled, followed by the white?

5. What is the probability that the black dice rolls 3 first?

1. In a **modified** game of 'Camel Up' only the blue camel is in play, only the blue die is placed in the pyramid, and during each leg there is only 1 die rolled. The initial position of the blue camel is 3.

(a) Fill in the probability distribution for the final position of the blue camel for the first leg of the race.

| Position ($p$) | 4 | 5 | 6 |
|---|---|---|---|
| $P(\text{Position} = p)$ | | | |

(b) Explain how this random event (the leg of the race) could be simulated using a 6 sided die.

(c) Simulate the game 20 times, and estimate the expected landing position for the camel.

| Simulation | Die | Position |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |
| 10 | | |
| 11 | | |
| 12 | | |
| 13 | | |
| 14 | | |
| 15 | | |
| 16 | | |
| 17 | | |
| 18 | | |
| 19 | | |
| 20 | | |

(d) What is the theoretical expected landing position for the blue camel?

2. In a **modified** game of 'Camel Up', there are three camels in play: Blue, Red, and Yellow. During each leg there are exactly two die rolled out of the pyramid. In this modified game, the camels are unable to stack on each other and may occupy the same space. This means tied camels are **both** in first place.

| Colour | Red | Blue | Yellow |
|---|---|---|---|
| Positions | 4 | 5 | 7 |

(a) Determine the marginal probability distribution for each of the camels final positions:

**Red**

| Position ($p$) | | | | |
|---|---|---|---|---|
| $P(\text{Position} = p)$ | | | | |

**Blue**

| Position ($p$) | | | | |
|---|---|---|---|---|
| $P(\text{Position} = p)$ | | | | |

**Yellow**

| Position ($p$) | | | | |
|---|---|---|---|---|
| $P(\text{Position} = p)$ | | | | |

(b) What are the expected position for each of the camels?

(c) What is the probability that blue ends the leg in first place?

3. In a **modified** game of 'Camel Up', there are three camels in play: Blue, Red, and White. There is a *regular* white dice used instead of the *crazy* dice. During each leg there are exactly two die rolled out of the pyramid. The initial positions and heights of the camels are shown below (height 1 refers to bottom of a stack):

| Colour | Red | Blue | White |
|---|---|---|---|
| Positions | 2 | 3 | 14 |
| Heights | 1 | 1 | 1 |

(a) Determine the theoretical probability distribution for each racing camel's final position at the end of the first leg:

**Blue**

| | | | | |
|---|---|---|---|---|
| Position ($p$) | | | | |
| $P(\text{Position} = p)$ | | | | |

**Red**

| | | | | | |
|---|---|---|---|---|---|
| Position ($p$) | | | | | |
| $P(\text{Position} = p)$ | | | | | |

(b) Discuss how the red camels final rank for the leg could be simulated using a three six sided die.

(c) Run 10 simulations to estimate the probability that red is in first place at the end of the leg.

| Simulation | Camel Order | First Camel Movement | Second Camel Movement | Final Rank |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |
| 7 | | | | |
| 8 | | | | |
| 9 | | | | |
| 10 | | | | |

(d) Determine the theoretical probability that the red camel is in first place at the end of the first leg. (Remember that white, the 'crazy camel' is not in the race, and thus can never win.)

4. In a modified game of 'Camel Up', players are not allowed to place any tokens, but the leg is played out as normal (with the normal dice). The initial positions and heights of the camels are shown below (height 1 refers to bottom of a stack):

| Colour | Red | Blue | Green | Purple | Yellow | White | Black |
|---|---|---|---|---|---|---|---|
| Position | 1 | 1 | 3 | 3 | 2 | 14 | 15 |
| Height | 1 | 2 | 1 | 2 | 1 | 1 | 1 |

(a) Write down all the possible final positions for the blue camel after the first leg.

(b) Discuss how blue's final rank after the first leg could be simulated using a single 6 sided dice.

(c) Run 2 simulations of the first leg for the blue camels final rank using the random numbers below (6 sided dice rolls)

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 3 | 4 | 1 | 6 | 5 | 3 | 5 | 6 | 1 | 4 | 6 | 3 | 6 | 3 |
| 3 | 4 | 3 | 6 | 1 | 4 | 3 | 2 | 5 | 3 | 5 | 6 | 1 | 2 | 1 |

(d) Using your 2 simulations, estimate the probability distribution for blue's rank.

(e) A python script is created that runs 10000 simulations for the first leg of the race with the indicated initial conditions. The estimated probability distributions for each colour's rank is shown below.

| | Red | Blue | Green | Purple | Yellow |
|---|---|---|---|---|---|
| Rank 1 | 0.0322 | 0.2776 | 0.0997 | 0.4502 | 0.1403 |
| Rank 2 | 0.1142 | 0.1470 | 0.3105 | 0.2671 | 0.1612 |
| Rank 3 | 0.1370 | 0.2736 | 0.2205 | 0.1673 | 0.2016 |
| Rank 4 | 0.2780 | 0.2668 | 0.1819 | 0.1001 | 0.1732 |
| Rank 5 | 0.4386 | 0.0350 | 0.1874 | 0.0153 | 0.3237 |

i. Going 'all in' for a round refers to the scenario where you obtain all the betting tickets for a particular camel. This would include one \$5 ticket, one \$3 ticket , and two \$2 tickets. Suppose you go 'all in' on red. What is your expected winnings/loss?

ii. For each camel, would it be better to roll all the 5 die (gaining \$5), or to go 'all in'.

5. In a **modified** game of 'Camel Up' each time a dice is rolled, it is reshuffled back into the pyramid. There are 4 dice in the pyramid (yellow, purple, green, and crazy), and a leg consists of 3 dice being rolled. The initial positions heights of the camels are shown below:

| Colour | Yellow | Purple | Green | White | Black |
|---|---|---|---|---|---|
| Position | 5 | 4 | 4 | 7 | 6 |
| Height | 1 | 2 | 1 | 1 | 1 |

(a) Over the course of the leg, what is the probability that the yellow dice rolls 2 exactly once?

(b) The *King of Yellow* rewards the kingdom with \$3, every time the yellow camel is rolled, and demands \$0.50 of taxation every time the yellow camel is not rolled. In 3 legs of the game, is the *King of Yellow's* expected income?

(c) What is the probability that the yellow camel moves 3 times in a row and moves a total distance of five?

(d) Every time the green dice is rolled the *Queen of Green* rewards herself with tea a biscuits. How many rolls are expected before the queen delights in tea and biscuits?

(e) What is the probability that the queen has 'tea a biscuits' for the third time on the $12^{th}$ roll of the dice? This could be over multiple legs.

(f) How many dice rolls would be expected for the queen to delight in 'tea a biscuits' for the fifth time? This could be over multiple legs.

(g) The *Lord of Rainbows* enjoys seeing exactly two regular dice are rolled during a leg. Whenever this happens, he treats himself to 3 cookies. How many cookies is he expected to eat in a leg?

(h) In one leg of the race, what is the probability that the *King of Yellow* rewards \$3 to the kingdom, the *Queen of Green* has tea and biscuits, and the *Lord of Rainbows* treats himself to 3 cookies?