Vector Geometry

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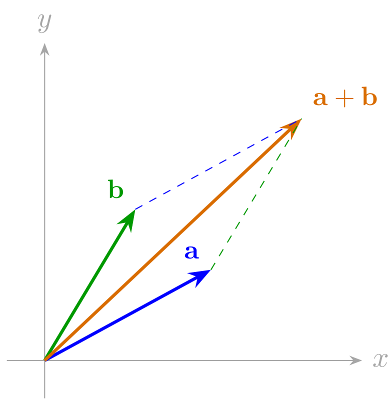
This short note illustrates three fundamental operations with vectors in $\mathbb{R}^2$

1. vector addition $\mathbf{a} + \mathbf{b}$;
2. vector subtraction $\mathbf{a} - \mathbf{b}$;
3. scalar multiplication $k\,\mathbf{v}$ (stretching/shrinking and direction).

1. Vector Addition

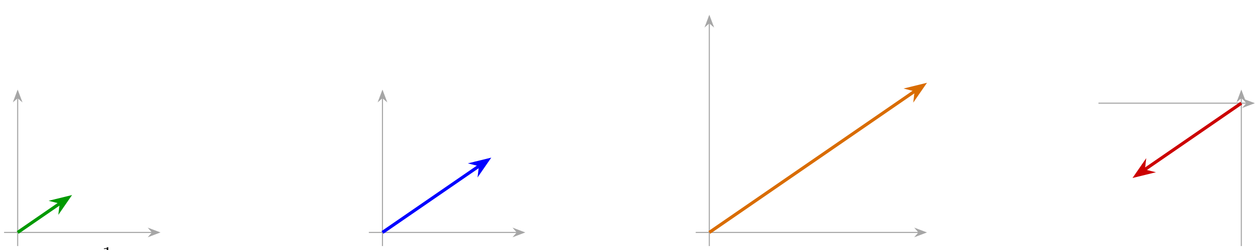
Rule. Place $\mathbf{b}$ with its tail at the head of $\mathbf{a}$; the sum $\mathbf{a} + \mathbf{b}$ is the arrow from the tail of $\mathbf{a}$ to the head of $\mathbf{b}$. Equivalently, it is the diagonal of the parallelogram formed by $\mathbf{a}$ and $\mathbf{b}$. Addition is commutative.



Notes: $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$. The parallelogram (dashed lines) shows both tip-to-tail paths leading to the same sum.

3. Scalar Multiplication

Rule. For a scalar $k \in \mathbb{R}$ and a vector $\mathbf{v}$: $\|k\mathbf{v}\| = |k|\,\|\mathbf{v}\|$; direction is the same as $\mathbf{v}$ if $k > 0$, reversed if $k < 0$; $k = 0$ gives the zero vector.



Notes: Stretching ($|k| > 1$) lengthens the arrow; shrinking ($0 < |k| < 1$) shortens it; a negative k flips its direction.

2. Vector Subtraction

Rule. $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$. Geometrically, you may reverse $\mathbf{b}$ and add it to the head of $\mathbf{a}$.