Assignment #1
September 2022

1. Kuttler section 1.2 problems 3, 5, 6, 12, 15, 19, 20, 27, 32, 46, 57, 58

2. Given the following system of linear equations:

$$\begin{aligned} -x - y &= -4 \\ 2x + 2y &= 8 \end{aligned}$$

   (a) Graph the set of solutions for each equation

   (b) Based on the graph how many solutions foes the system have?

3. Start with the following matrix:

$$\begin{bmatrix} 0 & 10 & -8 \\ 9 & 4 & 10 \\ -8 & -3 & -9 \end{bmatrix}$$

   Perform the following 3 elementary row operations, one after the other, and give the resulting matrix at each step.

   (a) add -2 times row 1 to row 2

   (b) Multiply row 3 by 7

   (c) Interchange rows 2 and 3

4. Solve the following system of linear equations

$$\begin{aligned} 5x_1 + 5x_2 - 10x_3 &= -15 \\ -3x_1 - 3x_2 + 6x_3 &= 9 \\ x_1 + x_2 - 2x_3 &= -3 \end{aligned}$$

   If the system has no solutions demonstrate this by giving a row-echelon form oof the augmented matrix for the system.

5. Solve the following system of linear equations

$$\begin{aligned} 3x_1 + 3x_2 - 9x_3 &= -3 \\ 2x_1 + 2x_2 - 5x_3 &= -1 \end{aligned}$$

   If the system has no solutions demonstrate this by giving a row-echelon form oof the augmented matrix for the system.

6. Solve the following system of linear equations

$$x_1 - 5x_2 - x_3 = 3$$

If the system has no solutions demonstrate this by giving a row-echelon form oof the augmented matrix for the system.

7. The reduced row-echelon form of the augmented matrix for a system of linear equations with variables $x1, \ldots, x6$ is given below. Determine the solutions for the system and enter them below.

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & -4 & -3 & 2 \\ 0 & 1 & 0 & 0 & 5 & 3 & 0 \\ 0 & 0 & 1 & 0 & -2 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 5 & 3 \end{array}\right]$$

8. Determine the values of a for which the following system of linear equations has no solution, a unique solution or infinitely many solutions. you can select 'always', 'never', $a =$ or $a \neq$, then specify a value or comma-separated list of values.

$$\begin{aligned} x_1 + ax_2 + 2x_3 &= 3 \\ 2x_1 - 4x_2 + 5x_3 &= 10 \\ 3x_1 - 6x_2 + 7x_3 &= 13 \end{aligned}$$

9. A quadratic function is a function of the form $y = ax^2 + bx + c$, where a, b and c are constants. Given any three points in the plane, there is exactly one quadratic function whose graph contains these points. Find the quadratic function whose graph contains the points $(0, -5)$, $(1, -4)$ and $(-1, 0)$.

10. Consider the following matrices:

$$A = \begin{bmatrix} 3 \\ 1 \\ -3 \\ -3 \end{bmatrix} \quad B = \begin{bmatrix} -6 \\ 0 \\ 7 \\ 5 \end{bmatrix} \quad C = \begin{bmatrix} -6 \\ 4 \\ 10 \\ 1 \end{bmatrix}$$

For each of the following matrices, determine whether it can be written as a linear combination of matrices A, B and C. If so, give the linear combination using the matrix names above.

$$V_1 = \begin{bmatrix} -6 \\ -6 \\ 11 \\ 0 \end{bmatrix} \quad V_2 = \begin{bmatrix} -18 \\ 2 \\ 23 \\ 12 \end{bmatrix} \quad V_3 = \begin{bmatrix} 4 \\ -2 \\ -9 \\ 7 \end{bmatrix}$$

11. Solve the following system of linear equations

$$\begin{aligned} 5x_3 + 10x_4 &= -10 \\ x_1 + 3x_2 - 2x_3 - 2x_4 &= -3 \\ x_3 + 2x_4 &= -1 \end{aligned}$$

If the system has no solutions demonstrate this b giving a row-echelon form of the augmented matrix for the system.