Vector Racing
Dr. Vince

1. Create a track on a grid paper. Ensure that there are grid intersections through the track (so avoid a tiny corridor that is in-between grid lines). (See additional rules for advanced variantions.)

2. Decide on:
   (a) a direction of travel,
   (b) an orientation for the coordinate system,
   (c) a start/finish line (which follows a grid line),
   (d) and a shared starting position for the racers.

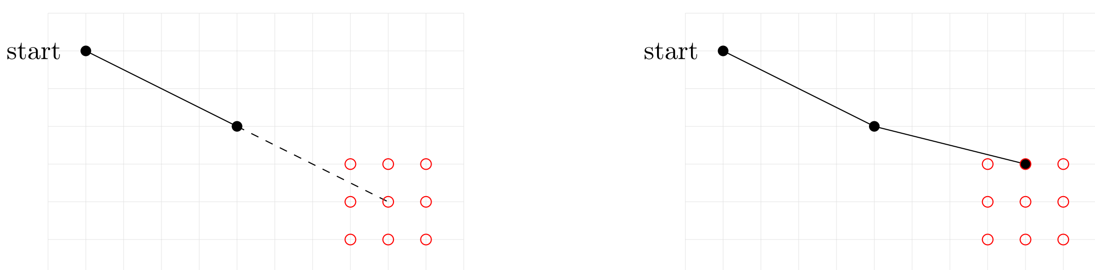
3. Each player should have a different colour and/or mark for their racer to distinguish them.

4. Every player writes down their move using the coordinate system, written as a pair of coordinates representing the change in x and the change in y. (For younger players, they may use R for right, L for left, U for up, and D for down instead of using coordinates. For example, $(-1, 3)$ is 1L3U.)

5. If a player's move is **valid**, put down that player's mark in the new location and join the previous position to the new position. A move is valid if:
   (a) The x-coordinate of the new move is within 1 of the x-coordinate of the old move.
   (b) The y-coordinate of the new move is within 1 of the y-coordinate of the old move.

   E.g. If player A's previous move was to move their racer $(4, -2)$, then their next move can be any one of $(3, -3), (4, -3), (5, -3), (3, -2), (4, -2), (5, -2), (3, -1), (4, -1), (5, -1)$.

   OR: If player A's previous move was to move their racer 4R2D, then their next move can be any one of 3R3D, 3R2D, 3R1D, 4R3D, 4R2D, 4R1D, 5R3D, 5R2D, 5R1D.



The racer has moved $(4, -1)$, or 4R1D.

   Note: A player's first move assumes their previous move was no motion at all, so valid moves are $(-1, -1), (-1, 0), (-1, 1), (0, -1), (0, 0), (0, 1), (1, -1), (1, 0), (1, 1)$.

6. If the line joining the old position to the new position touches or cross the boundary of the track at any point, the racer has crashed.
   (a) The car is moved to the point on the track nearest to where the racer first crashes, that is closer to the start line and further from the finish line.
   (b) The racer loses their next turn repairing their vehicle.
   (c) The racer begins turn after their lost turn with 0 velocity, as if they are just starting the race.
   (d) (See additional rules for advanced variantions.)

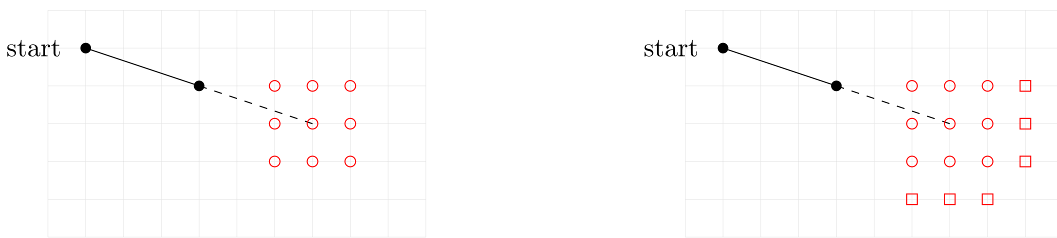
7. The first racer to cross the finish line wins. Since all racers move at the same time each turn, in the event that more than 1 racer crosses the finish line in a single turn, there is a tie. (See additional rules for advanced variantions.)

Vector Racing: Advanced variations

Track set-up:

You may choose to add additional aspects to the track, such as power-ups or hazards. Here are some examples:

1. Oil slick: if a chosen position would cause the racer to pass through an oil slick, their tires become slippery and their next move must be a copy of the previous move.
2. Speed up: on the next turn, the racer may move an additional **1 unit more in either direction**. For example, if the previous move was $(3,-1)$, the six new moves available in addition to the usual 9 moves are: $(5,-2),(5,-1),(5,0),(2,-3),(3,-3)$, or $(4,-3)$. Notice that $(1,-1),(3,1),(5,-2)$ are not allowed, for instance.

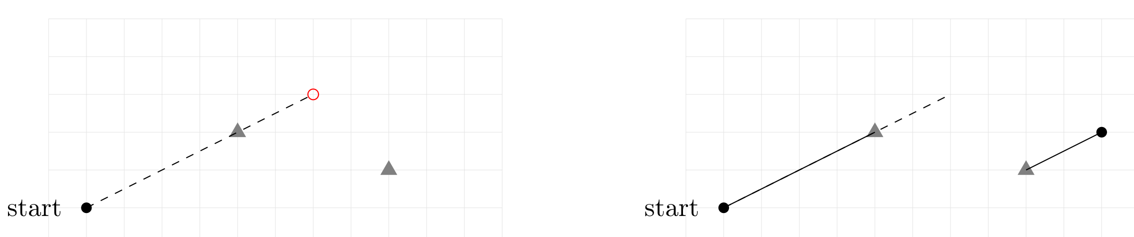


With a speed up, the squares become valid moves as well.

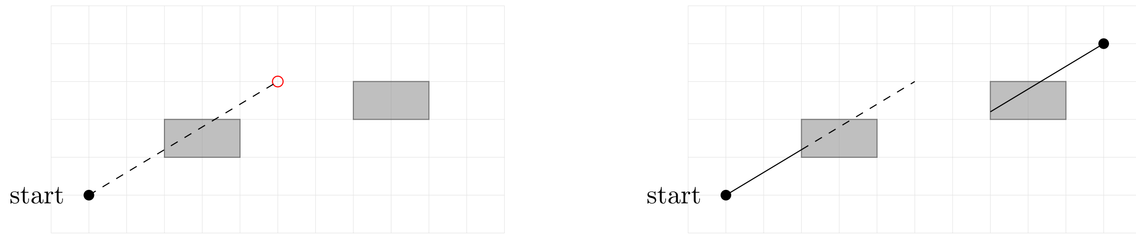
3. Slow down: on the next turn, the racer may move an additional **1 unit less in either direction**. For example, if the previous move was $(3,-1)$, the three new move available in addition to the usual 9 moves are: $(1,-1),(1,-2)$, or $(1,-3)$. Notice that $(2,1),(3,1),(4,1)$ are not allowed (the racer isn't slowing down in that case).

With a slow down, the squares become valid moves as well.

4. Warps: designate two points in the track to be connected by a wormhole. If a chosen position would cause the racer to pass through a war point, the car ends up exiting at the other warp point and continues its velocity. (Warps can be made to be points or larger regions.)



A racer attempts to move (6,3), passing through a warp point (the triangle) and ending at the circle.



A racer attempts to move (5,3), passing through a warp area (the shaded region) and ending at the circle.

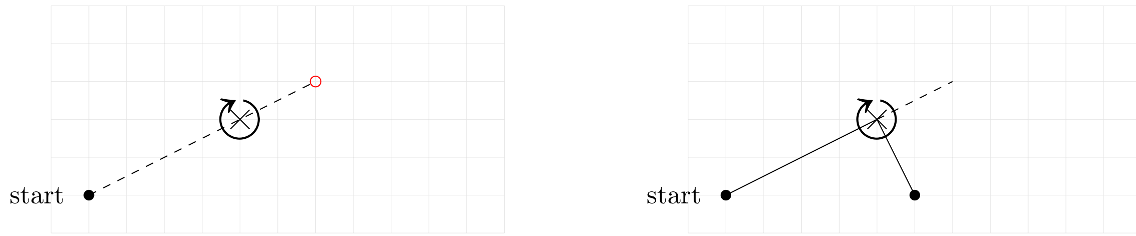
5. Reflectors: Reflectors are lines that are only on the grid lines (for ease of calculation). When a racer's path would normally go through a reflector, it instead gets reflected off the reflector with the angle of incidence equal to the angle of reflection and continues for the remainder of its velocity. (It is easiest to draw an imaginary point past the reflector, then reflect its image across the boundary.) For the next turn, if the desired move was (x, y), then treat the reflected move as if it were moved $(x, -y)$ (if the reflector is horizontal) or $(-x, y)$ (if the reflector is vertical).

A racer attempts to move (6,3), bouncing off a reflector (the thick line) and ending at the circle.

A racer attempts to move (5,3), bouncing off a reflector (the thick line) and ending at the circle.

6. Spinners (advanced): A spinner is a point only at intersections of the grid lines (for ease of calculation). When a racer's path would normally go through a spinner, it instead gets spun around 90 degrees at that point and continues for the remainder of its velocity. There are two types of spinners: clockwise and counter-clockwise. As the names suggest, clockwise spinners rotate the racer 90° clockwise, whereas counter-clockwise spinners rotate the racer 90° counter-clockwise. (It is easiest to draw an imaginary point past the spinner, then rotate its image around the spinner.) For the next turn, if the desired move was (x, y), then treat the rotated move as if it were moved $(y, -x)$ (if the spinner is clockwise) or $(-y, x)$ (if the spinner is counter-clockwise).

7. To add difficulty, create a requirement that racers must hit some number of these power-ups/hazards.



A racer attempts to move (6,3), running into a clockwise spinner (the cross, identified with the clockwise arrow) and ending at the circle.

A racer attempts to move (6,3), running into a counter-clockwise spinner (the cross, identified with the counter-clockwise arrow) and ending at the circle.

Crashing:

1. Decide what the damage does:
   (a) Option: Racers have 10 life points. Reaching 0 means they are out of the race for good. Crashing at any speed means they lose a turn, as per the usual rules.
   (b) Option: Racers lose $n + 1$ turns, where n is the amount of damage they received.
2. Decide how much damage you receive upon crashing:
   (a) Option: Damage is equal to the sum of the x and y components of the move.
   (b) Option: Damage is equal to the velocity of the move, that is, the square root of the sum of squares of the x and y components of the move. (Requires Pythagorean theorem.)
   (c) Option: Damage is equal to the component of their velocity which is perpendicular to the wall, rounded up. (Requires an understanding of vector components and some geometry/trigonometry.)

Crossing the finish line simultaneously:

1. Option 1: The racer who was closest to the finish line on the turn before crossing wins.
2. Option 2: The racer with the largest velocity on the turn crossing the finish line wins.
3. Option 3 (advanced): For each racer who crossed the finish line at the same time, perform the following computations:
   (a) Compute d, the distance from the previous position to the finish line, along the line of travel. This can be done by using Pythagorean theorem on the full line of travel, then scaled using similar triangles.
   (b) Compute v_i, the velocity of the path previous to crossing the finish line.
   (c) Compute v_f, the velocity of the path that crosses the finish line.
   (d) Based on these values, compute the time it took for the racer to finish the race, $t = 2d/(v_i + v_f)$. The racer with the smallest value of t wins.