Math 10 - Cumulative Project I - Introduction To Algorithms

This project will focus on the competencies *problem solving*, *technology*, and *communication*. The goal of the project is designing algorithms/programs/general solutions that could be used to complete each task. Some tasks are significantly more challenging than others. General solutions will be coded into programs via Python. For more information on python, check out the website https://www.python.org.

There are a variety of tasks that have been put together for you to solve. The Tasks focus on the concepts we have learned in the first two units: Measurement, and Algebra and Number. For the project you must achieve a total of 8 points. You must have *at least* 2 points from both the measurement, and algebra and number sections. The tasks have been scored as follows:

1 - Easy
2 - Medium
3 - Hard

For example you could complete complete four 2 point questions, or two 3 point questions and one 1 point question. Challenge questions are left as exercises, you **do not** need to complete them.

Tasks - Measurement

Painting A Room (1 Point)

You have decided to paint a room. You need to paint four walls. The paint you want to use costs $4 per square foot. The room that you're painting has a width of w feet, and length of l feet.

i. Write a general solution for the cost of painting the area in terms of w and l.

ii. Finish the python program that finds the cost of painting the area. It has been partially completed below:

```
print("This is a python program that determines how much it will cost to paint 4 walls, each
    with width w and length l. The paint costs 4 dollars per square foot.")

# input known values $
l =     # Input length of wall
w =     # Input Width of wall
p =     # Input Cost Of Paint
# Calculate Cost of painting the room (Remember there are 4 walls !)
c =
# Print Calculated Cost
print(       )
```

Volume Of A Pool (1 Point)

An engineer is designing a pool. If the pool has a length l, width w, and depth d.

i. Write a general solution for volume v in terms of l, w, and d.

ii. Write a python program that takes the dimensions of pool in **feet** and will calculate the volume of the pool in m^3, $feet^3$, and $inches^3$.

Converting Between Moles and Grams (1 Point)

In Science class, you have been determining the molar mass of different molecular compounds. Given g grams of a particular compound, $g \in \mathbb{Q}$, $g > 0$. Convert weight into m moles, $g \in \mathbb{Q}$, $g > 0$.

i. Write an algorithm/general solution that will take a molar mass of a compound (M) and a weight in grams (g), and convert the weight to moles (m).

ii. Write a python program that will take a molar mass of a compound (M) and a weight in grams (g), and convert the weight to moles (m).

Lighthouse Problem (2 Points)

A lighthouse is being built so that is can spread its light over an area of a m^2, where $a \in \mathbb{Q}$ and $a > 0$. What should the engineers of the lighthouse make the height h in **meters** if the light is to reach/cover an area of a m^2, and the maximum distance a beam of light can travel is l km, where $l \in \mathbb{Q}$ and $l > 0$?

i. Write a general solution for h, in terms of a, and l.

ii. Write a python program that will find the height of the lighthouse, given the area it covers and the length the light can travel.
Hint: You will need to import `math` in python, in order to use math.pi for π

Savings Problem (2 Points)

You are saving money to purchase an item. The item costs a dollars, where $a \in \mathbb{Q}$, $a > 0$. Inx your bank account you have s dollars, where $s \in \mathbb{Q}$, $s > 0$. At your current job you are making m dollars a month, where $m \in \mathbb{Q}$, $m > 0$. Write an algorithm that determines how long, in **years** it will take until you have enough money to purchase the item. A sample solution is shown below:

i. Write a general solution for l, the length in years it will take before you are able to purchase the item. Write your solution in terms of a, s, and m.

ii. Write a python program that will take the cost of the item a, the amount of money you have saved s, and your monthly income m, and give the length of time in years until you are able to purchase the item.

Kinetic Energy Of A Moving Object (2 Points)

The Kinetic Energy of a moving object is given by the equation: $E_k = \frac{1}{2}mv^2$. Energy using the unit $joules$. When defining a joule using mass and velocity:

$$1\ joule\ = 1kg * \left(\frac{m}{s}\right)^2$$

i. Write a general solution for v in **km/h** if you are given the mass of an object m in kg, and its kinetic energy E_k in $joules$.

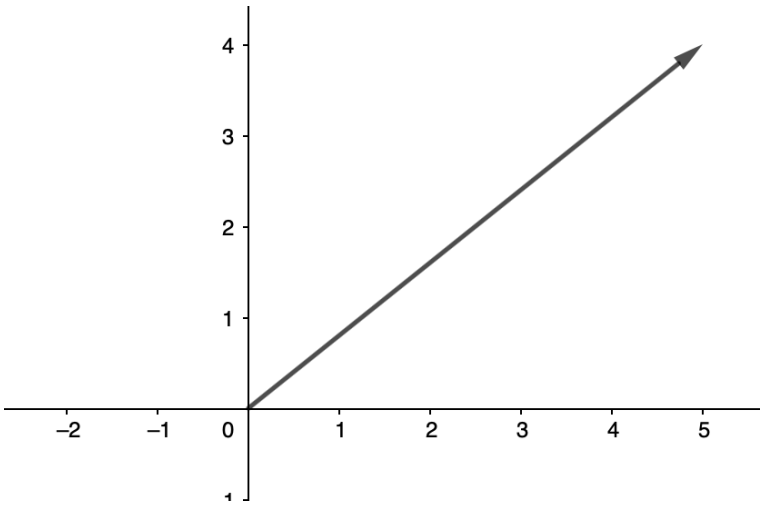
ii. Write a python program that will find the velocity of a moving object in **km/h** if you are given the mass of an object m and its kinetic energy E_k in $joules$.

Pythagorean Theorem In 3-Dimensions (3 Points)

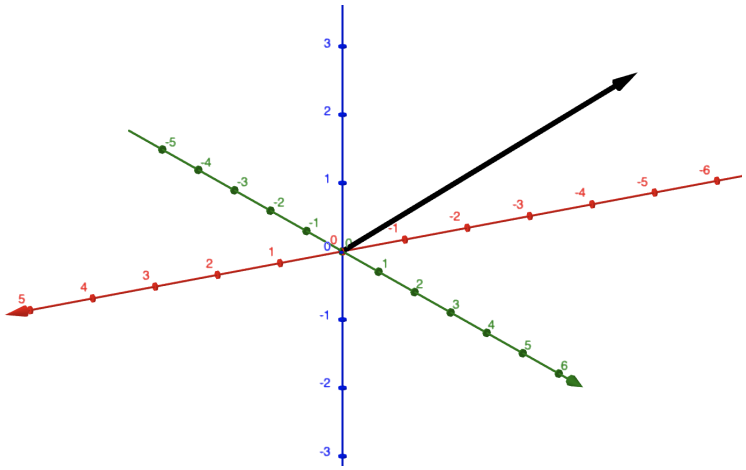
In your mathematics career, you have learned the **Pythagorean Theorem**:

$$a^2 + b^2 = c^2$$

This is often used to represent the hypotenuse of a right triangle, given the opposite and adjacent side. In other words, it represents a length of an arrow, given an x-coordinate, and a y-coordinate:



In this task, we would like to this formula to 3-dimensions. In other words, to find the length of an arrow in 3D, or $\mathbb{R}^3$:



i. Write a general formula for the length of the arrow, given its x, y, and z coordinates:

ii. Write a python program that will compute the length of an arrow in 3-dimensions, given its x, y, and z coordinates.
Challenge: Extend your formula to find the magnitude of an arrow/vector in $\mathbb{R}^n$.

Tasks - Algebra And Number

Pay It Forward (1 Point)

In the movie "Pay It Forward", a student, Trevor, comes up with an idea to make a positive impact on the world. He decides to do a good deed for three people and asks that each of the three people feel "pay it forward", doing a good deed for three more people. https://www.youtube.com/watch?v=F7Xnlc2GLfY.

Trevor's idea can be modelled by the equation $y = 3^x$, where y is the number of people that have been helped, and x is the current stage of the process.

i. What does the 3 represent in the equation?

ii. Finish this python program to determine how many people will have been touched by Trevor's ideas at stage x:

```
print("This is a python program that will:                                    ")

# input known values
s = int(input("input           : "))

# Make Calculation
y =

# Print Amount Of People That Have Been Touched
print(          )
```

What Is the Probability Of Winning? (2 Points)

When playing a certain game a person has probability p, $0 \leq p \leq 1$ of winning each round.

i. What is the probability of losing each round in the game?

ii. If the person plays the game for n rounds, what is the probability that they will only win on the n^{th} round? Write a general solution in terms of n and p.

iii. Write a program that takes the probability of winning a game, and determines the probability of winning the game only on the n^{th} round.

Challenge: If the person plays the game for n rounds, determine the probability that they will win once regardless of round. Write a general solution in terms of n and p.

Fission In A Nuclear Reactor (2 Points)

For a brief history on nuclear energy: https://www.youtube.com/watch?v=rcOFV4y5z8c

In a nuclear fission reaction, an atom is split by an accelerated neutron. The splitting of the atom causes a tremendous output of energy. Each atom that is split produces n new neutrons that will split n new atoms atoms. Every time a group of atoms is split, it is called a generation g.

i. In a nuclear fission reaction, write the general formula for the number of fissions that occurs in a nuclear reaction, given that each fission produces n neutrons, and there have been g generations.

ii. Write a python program that will take the number of neutrons produced by each fission n, the amount of generations, and outputs the number of fissions.

Magical Trap In Harry Potter (2 Points)

In *Harry Potter And The Deathly Hallows Part II*, there is a scene where Ron, Harry, and Hermione have broken into the magical bank Gringotts in search of a Horcrux: https://www.youtube.com/watch?v=mcMjdSk9EfY. The Gemino curse has been applied to the various treasure in the room. After touching a cup, it will multiple to become n cups.

i. A cup occupies v_{cup} m^3 of a room, and after being touched spontaneously multiplies to be n cups every 2 seconds. Write a general solution for how much volume will be occupied and t seconds.

ii. Write a python program that will take the multiplication factor n, the time t, and will output the volume that will be occupied by cups.

Challenge: If the room has a total volume of v_{room} m^3, how long will it take until the room is entirely full of cups? Write a general solutions for t in terms of v_{cup}, v_{room}, and n

Planetary Alignment Problem For Unknown Galaxy (3 Points)

In a solar system there are n planets, where $n \in \mathbb{N}$. Each planet has a unique orbital period T_i, $i = 1, 2, 3, \ldots, n$. If the planets are orbitally aligned at time t_o, design an algorithm to determine the time t when planets will align again. A sample solution is shown below:

i. Create a general algorithm that can be used to solve this problem for n different planets.

ii. Write a python script that will determine the next orbital alignment in a galaxy that has $n = 2$ planets.
Hint: You might want to use Python's gcd() function, that finds the greatest common factor of two numbers.

Challenge I: Write a python function, instead of using gcd.

Challenge II: Write a python script that will determine the next orbital alignment in a galaxy that has n planets.

The Locker Problem (3 Points)

In a school, n students are assigned n lockers. Students are assigned a number i, $i = 1, 2, 3, \ldots, n$, based on their locker number. The student assigned locker 1 opens all n lockers, the student assigned the number 2 closes all lockers that are multiples of 2. The i^{th} students opens/closes all lockers that are multiples of i. What lockers are open after the i^{th} student goes through the lockers?

i. What lockers will remain open after the n^{th} student passes through?

ii. Write a python script that will take the number of students n, and output the lockers that will remain open after the n^{th} student has passed through.

Iterative Approximation To A Radical (3 Points)

Given n, where $n \in \mathbb{W}$, we can approximate $\sqrt{2}$ using what is known as the *Babylonian Method.* First research the algorithm and write a brief explanation about how it works. Next, write a program that will carry out the algorithm, and approximate $\sqrt{2}$ with precision. A sample solution is shown below:

The Babylonian Method is used to approximate $\sqrt{a}$. The Algorithm uses the series:

$$x_{n+1} = \frac{x_o + \frac{S}{x_o}}{2}$$

For example, If I were to approximate $\sqrt{5}$:

i. Begin by guessing at what the number might be. We know that $2 < \sqrt{5} < 3$, so we will guess 2.5

ii. Next, we apply the Babylonian algorithm:

$$\begin{aligned} x_1 &= \frac{2.5 + \frac{5}{2.5}}{2} = 2.25 \\ x_2 &= \frac{2.25 + \frac{5}{2.25}}{2} = 2.236111 \\ x_3 &= \frac{2.236111 + \frac{5}{2.236111}}{2} = 2.236068 \\ x_3 &= \frac{2.236068 + \frac{5}{2.236068}}{2} = 2.236068 \end{aligned}$$

So $\sqrt{5} \approx 2.236068$.

i. Write a program that will approximate $\sqrt{a}$ using the Babylonian method.

ii. Research alternative algorithms that are used online, and write a brief description of another.