Two-Sample z Intervals for $p_1 - p_2$

AP Statistics · Mr. Merrick · February 9, 2026

We often want to compare two groups by estimating a difference in population proportions.

$$p_1 - p_2 = \text{(true proportion in Group 1)} - \text{(true proportion in Group 2)}$$

A confidence interval gives a range of plausible values for $p_1 - p_2$.

A two sample z-interval for $p_1 - p_2$ is given by:

$$(\hat{p}_1 - \hat{p}_2) \ \pm \ z^\star \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

- $\hat{p}_1 = \dfrac{x_1}{n_1}, \quad \hat{p}_2 = \dfrac{x_2}{n_2}$
- $z^\star = 1.645$ (90%), 1.96 (95%), 2.576 (99%)

Conditions

This method relies on the sampling distribution of the statistic $\hat{p}_1 - \hat{p}_2$ being approximately Normal and having a predictable standard deviation. The following conditions ensure that these assumptions are reasonable.

- **Random:** Random sampling or random assignment helps ensure that the samples are representative of their populations. Without randomness, the results may be biased, and the confidence interval may not reflect the true population difference.
- **Independence:** When sampling without replacement, the 10% condition ensures that the outcome of one observation does not meaningfully affect another. This allows us to treat observations as independent and use the standard error formula in the interval.
- **Large Counts (both groups):** Having at least 10 successes and 10 failures in each group ensures that the sampling distributions of $\hat{p}_1$ and $\hat{p}_2$ are approximately Normal. This, in turn, makes the distribution of their difference approximately Normal, which is required for the z interval.

If these conditions are satisfied, the two-sample z interval provides reliable results.

Example: Seatbelt use

A state compares seatbelt use between two regions.

- Urban region: $n_1 = 200$, $x_1 = 162$ wear seatbelts.
- Rural region: $n_2 = 180$, $x_2 = 126$ wear seatbelts.

Construct a 95% confidence interval for $p_1 - p_2$ (urban minus rural).

Step 1 (Parameter + target). Let $p_1 - p_2$ be the true difference in the proportions of all urban and rural drivers who wear seatbelts (urban minus rural). We will construct a 95% confidence interval for $p_1 - p_2$.

Step 2 (Conditions).
Random: the problem states that random samples were taken from both regions.
Independence: each sample is less than 10% of its region's driving population.
Normality: Urban: 162 wear seatbelts and 38 do not; Rural: 126 wear seatbelts and 54 do not. All counts are at least 10.
Since all conditions are met, the sampling distribution of $\hat{p}_1 - \hat{p}_2$ is approximately Normal, so a two-sample z interval is appropriate.

Step 3 (Compute). The point estimate for $p_1 - p_2$ is $\hat{p}_1 - \hat{p}_2$.

$$\hat{p}_1 = \frac{162}{200} = 0.810, \qquad \hat{p}_2 = \frac{126}{180} = 0.700$$

$$\hat{p}_1 - \hat{p}_2 = 0.110$$

The standard error is

$$SE = \sqrt{\frac{0.810(0.190)}{200} + \frac{0.700(0.300)}{180}} \approx 0.0440$$

For 95% confidence, the critical value is $z^\star = 1.96$.

$$ME = 1.96(0.0440) \approx 0.086$$

$$CI: \ 0.110 \pm 0.086 = (0.024, \ 0.196)$$

Step 4 (Interpret). We are 95% confident that the true difference in the proportions of urban and rural drivers who wear seatbelts (urban minus rural) is between 0.024 and 0.196.

Practice

Voter turnout

A researcher compares voter turnout in two age groups.

- Ages 18–29: $n_1 = 250$, $x_1 = 140$ voted.
- Ages 30–49: $n_2 = 220$, $x_2 = 154$ voted.

Construct and interpret a 95% confidence interval for $p_1 - p_2$ (18–29 minus 30–49).

Step 1 (Parameter + target). Let $p_1 - p_2$ be the true difference in voter turnout proportions for ages 18–29 and 30–49 (18–29 minus 30–49). We will construct a 95% confidence interval for $p_1 - p_2$.

Step 2 (Conditions).
Random: the problem states that random samples were taken from both age groups.
Independence: each sample is less than 10% of the population of registered voters in that age range.
Normality: Ages 18–29: 140 voted and 110 did not; Ages 30–49: 154 voted and 66 did not. All counts are at least 10.
Since all conditions are met, the sampling distribution of $\hat{p}_1 - \hat{p}_2$ is approximately Normal, so a two-sample z interval is appropriate.

Step 3 (Compute).

$$\hat{p}_1 = \frac{140}{250} = 0.560, \qquad \hat{p}_2 = \frac{154}{220} = 0.700$$

$$\hat{p}_1 - \hat{p}_2 = -0.140$$

$$SE \approx 0.0440, \quad ME = 1.96(0.0440) \approx 0.086$$

$$CI: \ (-0.226, \ -0.054)$$

Step 4 (Interpret). We are 95% confident that the true difference in voter turnout proportions (ages 18–29 minus ages 30–49) is between -0.226 and -0.054. Because the entire interval is negative, the data provide evidence that voter turnout is lower for ages 18–29.

Product defects

Two factories produce the same part.

- Factory A: $n_1 = 400$, $x_1 = 28$ defective.
- Factory B: $n_2 = 350$, $x_2 = 42$ defective.

Construct and interpret a 90% confidence interval for $p_1 - p_2$ (A minus B).

Step 1 (Parameter + target). Let $p_1 - p_2$ be the true difference in defect proportions for Factory A and Factory B (A minus B). We will construct a 90% confidence interval for $p_1 - p_2$.

Step 2 (Conditions).
Random: parts are randomly inspected from each factory's production.
Independence: each sample represents less than 10% of total production.
Large Counts: Factory A: 28 defective and 372 non-defective; Factory B: 42 defective and 308 non-defective. All counts are at least 10.
Since all conditions are met, the sampling distribution of $\hat{p}_1 - \hat{p}_2$ is approximately Normal, so a two-sample z interval is appropriate.

Step 3 (Compute).

$$\hat{p}_1 = \frac{28}{400} = 0.070, \qquad \hat{p}_2 = \frac{42}{350} = 0.120$$

$$\hat{p}_1 - \hat{p}_2 = -0.050$$

$$SE \approx 0.0216, \quad ME = 1.645(0.0216) \approx 0.036$$

$$CI: \ (-0.086, \ -0.014)$$

Step 4 (Interpret). We are 90% confident that the true difference in defect proportions (Factory A minus Factory B) is between -0.086 and -0.014. Because the entire interval is negative, the data suggest that Factory A has a lower defect rate.

What does it mean if 0 is inside the interval?

If a confidence interval for $p_1 - p_2$ contains 0, then a true difference of 0 is a plausible value for $p_1 - p_2$ at that confidence level. This means the data do not provide convincing statistical evidence of a difference between the two population proportions.