Hypothesis Testing Errors: Type I, Type II, and Power

AP Statistics · Mr. Merrick · February 4, 2026

In hypothesis testing, we make a decision (reject or fail to reject H_0), but reality could be different (H_0 might be true or false). This packet focuses on two possible mistakes:

$$\alpha = P(\text{reject } H_0 \mid H_0 \text{ true}) \qquad \beta = P(\text{fail to reject } H_0 \mid H_0 \text{ false})$$

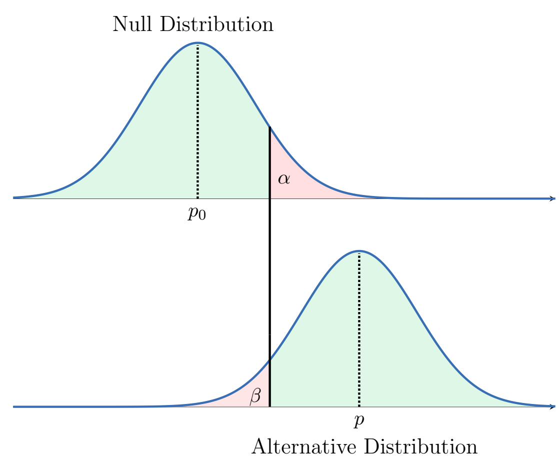
and on **power**, the probability we correctly reject H_0 when H_0 is false:

$$\text{power} = 1 - \beta.$$

Decision vs. Reality

- In a hypothesis test, we choose a rule that tells us when to reject H_0 (a *cutoff*).
- If H_0 is true, rejecting it is a Type I error (probability α).
- If H_0 is false, failing to reject it is a Type II error (probability β).
- When H_0 is false, the chance we correctly reject is power $= 1 - \beta$.

Reality \ **Decision**	Fail to Reject (FTR) H_0	Reject H_0
H_0 true	✓	**Type I error** $\alpha = P(\text{reject } H_0 \mid H_0 \text{ true})$
H_0 false	**Type II error** $\beta = P(\text{FTR } H_0 \mid H_0 \text{ false})$	✓



About the decision

- Rejecting H_0 does *not* prove H_a. It means the result would be unusually rare if H_0 were true.
- Failing to reject H_0 does *not* mean H_0 is true. It means the data is reasonably consistent with the null model.

How changes affect α, β, and power

Tradeoff (same sample size, same effect size)

- If you make α smaller (stricter cutoff), β usually gets larger (harder to reject).
- If you make α larger (easier cutoff), β usually gets smaller.

How to increase power

- Increase the sample size n (curves get narrower).
- Have a larger effect size (means are farther apart).
- Use a larger α (move the cutoff left), but this increases Type I risk.

Think of the cutoff as the line between "Fail to Reject" and "Reject." Moving the cutoff changes the size of the shaded regions.

- Make the cutoff *harder* to reach (more extreme to reject):

$$\alpha \downarrow \quad \beta \uparrow \quad \text{power } (1-\beta) \downarrow$$

- Make the cutoff *easier* to reach (less extreme to reject):

$$\alpha \uparrow \quad \beta \downarrow \quad \text{power } (1-\beta) \uparrow$$

Connection to p-values: choosing a smaller α means requiring a smaller p-value to reject H_0.

When is a Type I error worse vs. a Type II error worse?

Whether Type I or Type II errors are "more favorable" depends on the context (the consequences).

When Type I errors are worse (so we prefer a smaller α):

- Court trials: convicting an innocent person.
- Medical approval: approving a medication that does not actually work (or has no real benefit).
- Accusations/cheating investigations: punishing someone who didn't do it.

When Type II errors are worse (so we prefer smaller β / higher power):

- Disease screening: missing a real disease (sending a sick person home as "healthy").
- Safety systems: failing to detect a real problem (fire alarm, carbon monoxide detector).
- Quality control: shipping defective products as "good."

AP-style practice: interpreting errors

Problem 1: Airport Security Screening

An airport uses a screening test to detect prohibited items. Let H_0 be "a passenger does not have a prohibited item" and H_a be "a passenger has a prohibited item."

1. Describe a Type I error in context.

2. Describe a Type II error in context.

3. Which error would the airport consider more serious? Explain.

Solution.

1. **Type I error (reject H_0 when H_0 is true):** The screening test flags a passenger even though the passenger does *not* have a prohibited item (a false alarm).

2. **Type II error (fail to reject H_0 when H_0 is false):** The screening test does *not* flag a passenger who actually *does* have a prohibited item (a miss).

3. **Which is more serious?** Usually a Type II error is more serious for safety because a real threat is missed. However, the best answer depends on consequences and priorities (safety vs. cost/time/inconvenience).

Problem 2: New Tutoring Program

A school claims a new tutoring program increases the pass rate on a final exam above 70%. Let p be the true pass rate for students in the program.

$$H_0 : p = 0.70 \qquad H_a : p > 0.70$$

1. Interpret $\alpha = P(\text{reject } H_0 \mid H_0 \text{ true})$ in context.

2. Interpret $\beta = P(\text{fail to reject } H_0 \mid H_0 \text{ false})$ in context.

3. Interpret power in context.

Solution.

1. α: The probability we conclude the tutoring program increases the pass rate above 70% (reject H_0) even though, in reality, the true pass rate is $p = 0.70$.

2. β: The probability we fail to conclude the tutoring program increases the pass rate above 70% (fail to reject H_0) even though, in reality, the true pass rate is actually greater than 70% (that is, $p > 0.70$).

3. **Power:** The probability we correctly conclude the tutoring program increases the pass rate above 70% (reject H_0) when the program truly does increase the pass rate (that is, when $p > 0.70$). Equivalently, power $= 1 - \beta$.