Matched Pairs t Intervals and Tests for a Mean Difference

AP Statistics · Mr. Merrick · February 22, 2026

In matched pairs designs (before/after measurements or closely matched subjects), we analyze the mean difference. For each pair, compute a difference:

$$d_i = (\text{measurement 1}) - (\text{measurement 2})$$

We then treat the list of differences as one sample and perform a one-sample t procedure on the parameter

$$\mu_d = \text{true mean difference.}$$

Important: A matched pairs procedure is *not* a two-sample procedure. We reduce the data to one list of differences first.

1) Matched Pairs t Confidence Interval

Parameter: μ_d = true mean difference.

Check conditions:

- **Random:** The matched pairs are obtained from a random sample or from random assignment in a matched pairs experiment.
- **Independence:** The pairs are independent of one another (if sampling without replacement, check $n \leq 0.10N$).
- **Normal/Large Sample:**
  - If $n < 30$: the distribution of differences is roughly symmetric with no outliers.
  - If $n \geq 30$: CLT supports Normality of $\bar{d}$.

If conditions are satisfied, the interval is

$$\boxed{\bar{d} \pm t^{\star} \frac{s_d}{\sqrt{n}}} \quad \text{with } df = n - 1.$$

2) Matched Pairs t Hypothesis Test

Parameter: μ_d = true mean difference.

Typical null: $H_0 : \mu_d = 0$

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- **Normal/Large Sample:**
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Test statistic:

$$\boxed{T = \frac{\bar{d} - \mu_{d,0}}{s_d/\sqrt{n}}} \quad \text{with } df = n - 1.$$

Under H_0, T follows a t distribution with $n - 1$ degrees of freedom.

Example 1

A fitness program measures resting heart rate (beats per minute) before and after a 6-week training program for a random sample of 12 participants. Differences are computed as:

$$d = (\text{Before}) - (\text{After})$$

Summary statistics of the differences:

$$n = 12, \qquad \bar{d} = 4.5, \qquad s_d = 3.2$$

Construct and interpret a 95% confidence interval for μ_d.

Example 2

Students take a practice exam before and after a review session. Differences are defined as

$$d = (\text{After}) - (\text{Before})$$

Summary statistics:

$$n = 18, \qquad \bar{d} = 3.1, \qquad s_d = 4.5$$

Test, at the $\alpha = 0.05$ level, whether the review session improves scores on average.