Matched Pairs t Intervals and Tests for a Mean Difference

AP Statistics · Mr. Merrick · February 22, 2026

In matched pairs designs (before/after measurements or closely matched subjects), we analyze the mean difference. For each pair, compute a difference:

$$d_i = (\text{measurement 1}) - (\text{measurement 2})$$

We then treat the list of differences as one sample and perform a one-sample t procedure on the parameter

$$\mu_d = \text{true mean difference.}$$

Important: A matched pairs procedure is *not* a two-sample procedure. We reduce the data to one list of differences first.

1) Matched Pairs t Confidence Interval

Parameter: μ_d = true mean difference.

Check conditions:

- **Random:** The matched pairs are obtained from a random sample or from random assignment in a matched pairs experiment.
- **Independence:** The pairs are independent of one another (if sampling without replacement, check $n \le 0.10N$).
- **Normal/Large Sample:**
  - If $n < 30$: the distribution of differences is roughly symmetric with no outliers.
  - If $n \ge 30$: CLT supports Normality of $\bar{d}$.

If conditions are satisfied, the interval is

$$\boxed{\bar{d} \pm t^\star \frac{s_d}{\sqrt{n}}} \quad \text{with } df = n - 1.$$

2) Matched Pairs t Hypothesis Test

Parameter: μ_d = true mean difference.

Typical null: $H_0 : \mu_d = 0$

Check conditions:

- **Random:** The matched pairs are obtained from a random sample or from random assignment in a matched pairs experiment.
- **Independence:** The pairs are independent of one another (if sampling without replacement, check $n \le 0.10N$).
- **Normal/Large Sample:**
  - If $n < 30$: the distribution of differences is roughly symmetric with no outliers.
  - If $n \ge 30$: CLT supports Normality of $\bar{d}$.

Test statistic:

$$\boxed{T = \frac{\bar{d} - \mu_{d,0}}{s_d/\sqrt{n}}} \quad \text{with } df = n - 1.$$

Under H_0, T follows a t distribution with $n - 1$ degrees of freedom.

Example 1

A fitness program measures resting heart rate (beats per minute) before and after a 6-week training program for a random sample of 12 participants. Differences are computed as:

$$d = (\text{Before}) - (\text{After})$$

Summary statistics of the differences:

$$n = 12, \qquad \bar{d} = 4.5, \qquad s_d = 3.2$$

Construct and interpret a 95% confidence interval for μ_d.

Solution.

Step 1 — State

Let μ_d be the true mean difference in resting heart rate (Before – After) for all similar participants. We will construct a 95% confidence interval for μ_d using a matched pairs t interval.

Step 2 — Justify

We will use a matched pairs t interval.

- **Random:** The problem states that the 12 participants were randomly selected.
- **Independence:** The participants are different individuals, so the matched pairs are independent. Because the population of potential participants is much larger than 12, it is reasonable that $n = 12 \leq 0.10N$.
- **Normal/Large Sample:** The sample size is small ($n = 12$), so we must examine the distribution of the differences. Assuming a histogram/boxplot shows the differences are roughly symmetric with no outliers, the Normality condition is reasonable.

Since conditions are met, a matched pairs t interval is appropriate.

Step 3 — Carry Out

$$\bar{d} \pm t^\star \frac{s_d}{\sqrt{n}}$$

Degrees of freedom:

$$df = n - 1 = 11$$

For 95% confidence, $t^\star = t_{0.975,11} \approx 2.201$.

$$SE = \frac{3.2}{\sqrt{12}} = \frac{3.2}{3.464} \approx 0.924$$

$$ME = 2.201(0.924) \approx 2.03$$

$$\text{CI: } 4.5 \pm 2.03 = (2.47,\ 6.53)$$

Step 4 — Interpret

We are 95% confident that the true mean reduction in resting heart rate is between 2.47 and 6.53 beats per minute.

Because the entire interval is positive, the program appears to reduce resting heart rate on average.

Example 2

Students take a practice exam before and after a review session. Differences are defined as

$$d = (\text{After}) - (\text{Before})$$

Summary statistics:

$$n = 18, \qquad \bar{d} = 3.1, \qquad s_d = 4.5$$

Test, at the $\alpha = 0.05$ level, whether the review session improves scores on average.

Solution.

Step 1 — State

Let μ_d be the true mean difference in scores (After – Before).

We will test:

$$H_0 : \mu_d = 0 \qquad \text{vs.} \qquad H_a : \mu_d > 0.$$

Step 2 — Justify

We will use a matched pairs t test.

- **Random:** The students were randomly selected.
- **Independence:** Each student forms one matched pair, and students are independent of one another. Because the number of students is small relative to the population, $n = 18 \leq 0.10N$ is reasonable.
- **Normal/Large Sample:** The sample size is moderate ($n = 18 < 30$). We would check that the distribution of differences is roughly symmetric with no outliers. Assuming this is true, Normality is reasonable.

Step 3 — Carry Out

$$T = \frac{\bar{d} - 0}{s_d/\sqrt{n}}$$

$$SE = \frac{4.5}{\sqrt{18}} = \frac{4.5}{4.243} \approx 1.06$$

$$T = \frac{3.1}{1.06} \approx 2.92$$

Degrees of freedom:

$$df = 17$$

Right-tailed p-value:

$$p = P(t_{17} \geq 2.92) \approx 0.005.$$

Plain-language meaning of the p-value

Assuming the true mean difference is 0, there is about a 0.5% probability of getting a sample mean difference of 3.1 points or higher just by random chance (in a sample of 18 students).

Step 4 — Conclude

Because $p = 0.005 < \alpha = 0.05$, we reject H_0.

There is convincing evidence that the review session increases scores on average.