# Sample Size for Estimating a Population Mean

*AP Statistics · Mr. Merrick · February 21, 2026*

When planning a confidence interval for a population mean $\mu$, we often want to know how large a sample is needed to guarantee a desired margin of error.

## Margin of error for a mean

For a one-sample confidence interval for a population mean using a known (or planned) standard deviation $\sigma$, the margin of error is

$$\text{ME} = \text{critical value} \cdot \frac{\sigma}{\sqrt{n}}$$

If the population standard deviation $\sigma$ is known (or we are planning using an estimate of it), we use a $z^\star$ critical value:

$$\text{ME} = z^\star \frac{\sigma}{\sqrt{n}}$$

To guarantee a margin of error no larger than a chosen value ME, solve for $n$:

$$\text{ME} = z^\star \frac{\sigma}{\sqrt{n}} \Rightarrow \sqrt{n} = \frac{z^\star \sigma}{\text{ME}} \Rightarrow n = \left(\frac{z^\star \sigma}{\text{ME}}\right)^2$$

Always round **up** to the nearest whole number.

**Explainer**

In practice, we nearly always use a $z$ critical value when planning sample size for a mean — even if we will ultimately use a $t$ interval — because the required $t^\star$ depends on $n$, which we are trying to determine.

Using $z^\star$ gives a very good approximation and slightly overestimates the needed sample size for small $n$, making the plan safe

## What if $\sigma$ is unknown?

If $\sigma$ is unknown, we must use a **planning estimate** for the population standard deviation.

Common choices include:

- A previous study's sample standard deviation
- A pilot study
- An educated guess based on experience

The formula remains:

$$n = \left(\frac{z^\star \sigma}{\text{ME}}\right)^2$$

There is no "most conservative" universal choice like $p = 0.50$ for proportions. The required sample size grows as $\sigma$ increases.

## Practice

### 1. Known standard deviation

A soda company wants to estimate the mean amount of soda in its cans. Suppose $\sigma = 0.12$ ounces. They want a 95% confidence interval with ME $\leq 0.02$. Find the required sample size.

$$n = \left(\frac{(1.96)(0.12)}{0.02}\right)^2 = \left(\frac{0.2352}{0.02}\right)^2 = (11.76)^2 = 138.2976 \Rightarrow n = 139$$

### 2. Using a prior estimate of $\sigma$

A previous study found the standard deviation of commute times to be about 8 minutes. Find the required sample size for a 90% confidence interval with ME $\leq 2$ minutes.

$$n = \left(\frac{(1.645)(8)}{2}\right)^2 = \left(\frac{13.16}{2}\right)^2 = (6.58)^2 = 43.2964 \Rightarrow n = 44$$

### 3. Comparing different variability levels

A manufacturer wants a 99% confidence interval with ME $\leq 1.5$ units.

1. Find $n$ if $\sigma = 4$.
2. Find $n$ if $\sigma = 7$.

$$n_{\sigma=4} = \left(\frac{(2.576)(4)}{1.5}\right)^2 = \left(\frac{10.304}{1.5}\right)^2 = (6.8693)^2 = 47.17 \Rightarrow n = 48$$

$$n_{\sigma=7} = \left(\frac{(2.576)(7)}{1.5}\right)^2 = \left(\frac{18.032}{1.5}\right)^2 = (12.0213)^2 = 144.51 \Rightarrow n = 145$$