One-Sample z and t Tests for a Population Mean

AP Statistics · Mr. Merrick · February 21, 2026

Assume we have a random sample of size n from a population with an unknown distribution. We want to test a claim about the parameter μ (the population mean).

We compute $\overline{x}$ and s from the sample. Sometimes the population standard deviation σ is known. When σ is unknown (most common), we estimate it using s.

z-test for μ (when σ is known)

Step 1 — State hypotheses

$$H_0 : \mu = \mu_0$$

$$H_a : \mu < \mu_0 \quad \text{or} \quad \mu > \mu_0 \quad \text{or} \quad \mu \neq \mu_0$$

Step 2 — Check conditions

- Random: random sample or randomized experiment.
- Independence: $n \leq 0.10N$ if sampling without replacement.
- Normal/Large Sample: population Normal, or n large enough for CLT.

If conditions are satisfied, then under H_0,

$$Z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}} \sim N(0, 1).$$

Step 3 — Compute the p-value
Match to H_a:

- Left-tailed ($\mu < \mu_0$): $p = P(Z \leq z)$
- Right-tailed ($\mu > \mu_0$): $p = P(Z \geq z)$
- Two-sided ($\mu \neq \mu_0$): $p = 2P(Z \geq |z|)$

Step 4 — Conclude
At significance level α:

- If $p \leq \alpha$, reject H_0.
- If $p > \alpha$, fail to reject H_0.

$$\boxed{Z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}}}$$

t-test for μ (when σ is unknown)

Step 1 — State hypotheses

$$H_0 : \mu = \mu_0$$

$$H_a : \mu < \mu_0 \quad \text{or} \quad \mu > \mu_0 \quad \text{or} \quad \mu \neq \mu_0$$

Step 2 — Check conditions

- Random: random sample or randomized experiment.
- Independence: $n \leq 0.10N$ if sampling without replacement.
- Normal/Large Sample: if n is small, sample data are roughly symmetric with no outliers.

When σ is unknown, we estimate it with s. Under H_0,

$$T = \frac{\overline{x} - \mu_0}{s/\sqrt{n}} \sim t_{n-1}.$$

Step 3 — Compute the p-value
Match to H_a (use $df = n - 1$):

- Left-tailed: $p = P(T \leq t)$
- Right-tailed: $p = P(T \geq t)$
- Two-sided: $p = 2P(T \geq |t|)$

Step 4 — Conclude
At significance level α:

- If $p \leq \alpha$, reject H_0.
- If $p > \alpha$, fail to reject H_0.

$$\boxed{T = \frac{\overline{x} - \mu_0}{s/\sqrt{n}}, \quad df = n - 1}$$

When n is small and the population distribution is unknown

The t-test requires that the population distribution be approximately Normal when n is small. In practice, because we do not see the entire population, we examine the sample data. If the sample distribution is roughly symmetric with no strong skewness or outliers, it is reasonable to proceed with a t-test.

In AP Statistics, when n is small, we examine the sample data directly. If the sample distribution is:

- roughly symmetric,
- free of strong skewness,
- and free of outliers,

then it is reasonable to proceed with a t-test.

If the sample shows strong skewness or clear outliers and n is small, the t-test may not be reliable.

AP Exam Tip

When n is small, and the population isn't stated as normal, describe the sample distribution. If a graph is provided, reference it directly in your explanation. If raw data are given, use your calculator to create a graph (such as a histogram), then describe the shape.

A strong AP response includes words such as:

- roughly symmetric
- no obvious outliers
- not strongly skewed

Do not simply write "the calculator says it is Normal." Always justify using the shape of the distribution.

Example 1

A company that manufactures protein bars claims that the mean protein content is 20 grams per bar. A nutrition inspector suspects the bars may contain less protein than advertised. A random sample of 15 bars is tested. The sample mean protein content is 19.2 grams with a standard deviation of 1.5 grams. Does the sample provide convincing evidence at the $\alpha = 0.05$ level that the true mean protein content is less than 20 grams per bar?

(a) State the appropriate hypotheses and define the parameter.

(b) Check the conditions for inference.

(c) Perform the test at the $\alpha = 0.05$ level.

(d) Interpret the p-value in context.

(e) Explain what a Type I error would mean in this context.

(f) Explain what a Type II error would mean in this context.

(g) Describe one way to increase the power of this test.

Example 2

A bottling plant fills soda cans with 12 ounces of soda. The known process standard deviation is $\sigma = 0.25$ ounces. A random sample of 36 cans has a mean of 12.09 ounces. Does the sample provide convincing evidence at the $\alpha = 0.01$ level that the machine is overfilling soda cans?

(a) State hypotheses.

(b) Check conditions.

(c) Perform the test at $\alpha = 0.01$.

(d) Interpret the conclusion in context.

(e) Define a Type I error in context.

(f) Suppose the true mean is 12.15 ounces. Explain what power represents.

Example 3

A pharmaceutical company claims that a new medication results in an average recovery time of 8 days. A hospital study of 20 patients finds a mean recovery time of 9.1 days with standard deviation 2.4 days. Does the sample provide convincing evidence at the $\alpha = 0.05$ level that the true mean recovery time differs from 8 days?

(a) State hypotheses.

(b) Perform the test at $\alpha = 0.05$.

(c) State the conclusion in context.

(d) Describe a Type II error in context.

(e) If the true mean were 9.5 days instead of 8.5 days, would power be higher or lower? Explain.

Example 4

A school district claims the average SAT math score is 520. A random sample of 25 students has mean 508 and standard deviation 40. Does the sample provide convincing evidence at the $\alpha = 0.05$ level that the true mean SAT math score is below 520?

(a) State hypotheses.

(b) Conduct the test at $\alpha = 0.05$.

(c) Interpret the conclusion in context.

(d) What is the probability of committing a Type I error?

(e) Suppose the true mean is 500. Explain what power represents.

(f) Describe two ways to increase the power of this test.